Cost of Financial Options and Guarantees for an Interest Sensitive Annuity product in Taiwan under a Market Consistent framework

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1 Introduction

1.1 This report values an insurance product on the market-consistent framework.

1.2 Section 2 outlines the methodology used while Section 3 discusses the asset calibrations. Section 4 is the results and conclusions. The limitations and suggestions are shown in Section 5. The EEV Principles are shown in Appendix A. Appendix B summarises the product features while Appendix C lists the assumptions used in valuing the liabilities. Appendix D summarises the Economic Scenario Generator ("ESG") model.

1.3 The traditional embedded value has recently been criticised for a number of points including the lack of consistency in methodology, assumptions and disclosure practices, and the approach taken to value guarantees and options. In order to address this, a number of leading European insurers have grouped together to jointly release a document called the "European Embedded Value Principles". The document is fairly short and follows a principles-based rather than a rules-based approach. The high-level wording ensures that it covers the breadth and depth of the current Embedded Value ("EV') methodologies used by the member companies, although it does restrict current practice in some areas. The CFO Forum also produced another document, "Basis for Conclusions", in which it sets out some of the rationale behind the principles.

1.4 Many European companies have adopted these principles and are now calculating what is called European Embedded Values ("EEV")

1.5 The EEV framework has introduced an important concept which has largely been excluded in the traditional EV methodology, namely the cost of financial options and guarantees. It suggests that this cost should be deducted from the company's embedded value in order to reflect the cost of offering life insurance products with underlying guarantees, such as return of capital or guaranteed minimum annuity benefit and guaranteed minimum death benefit.

1.6 In practice, there are two approaches in the calculation of the cost of Financial Options and Guarantees ("FOG"), specifically market-consistent (or risk neutral) and real world. Under the market-consistent framework, option valuation is calibrated to reproduce market traded prices. As for the real world framework, option valuation is calibrated to reflect management views of the expected level of future asset values and return.

Policyholder interactions

1.7 The purpose of this report is to demonstrate some potential complications in calculating the cost of the FOG. This will involve the concept of the cost of the FOG and liability which are fully explained in Section 2 and asset modelling in Section 3. In addition, this report examines the impact on cost of the FOG using two different models, one excluding dynamic policyholder interactions and the other including dynamic policyholder interactions.

1.8 The model that excludes the dynamic policyholder interactions assumes that the policyholder is indifferent in all economic scenarios on surrender and as a result a deterministic discontinuance rate is assumed.

1.9 For the model that includes dynamic policyholder interactions, it is assumed that policyholders are rational and always exercise their right to surrender at the best timing in order to optimise the insurance contract's financial value to them. This model applies a stochastic discontinuance rate which is closely correlated to the market interest rate.

1.10 In this report, by comparing the results from the two models, we analyse the impact on the cost of the FOG by allowing for dynamic policyholder interactions in our modelling.

Product

1.11 A Taiwanese single premium interest sensitive annuity product is selected for this project. Many products in Taiwan that were issued in the last few decades offered policyholders high guaranteed crediting rates. In the recent low interest rate environment, these unsupportable high crediting rate products have led to insurers having significant interest risk exposure. As a consequence, many companies, such as ING and Prudential (UK), needed to inject substantial capital in order to bring their solvency ratios up to international standards.

Modelling

1.12 In order to set up a liability model that derives each of the cash flow items of the product, Watson Wyatt's in-house program, VIP 4.5, has been utilised as the modelling tool.

1.13 Watson Wyatt's in-house Market Consistent Asset Model ("MCAM") is the ESG that has also been used to produce an arbitrage free and market consistent economic scenario file. The economic scenario file is calibrated to Taiwanese assets as the product is denominated in Taiwan dollars ("TWD"). The asset model then generates 10,000 different scenarios from the economic scenario file, with each scenario carrying equal weight.

1.14 I would like to express my gratitude to Watson Wyatt Worldwide for the use of both of its proprietary actuarial software systems, Watson Wyatt VIP system and MCAM, in conducting this project. I would also like to extend my appreciation to Watson Wyatt's asset team in the Reigate office, especially Mr. Zhang Fei Fei in helping me to gain a deeper understanding of the market-consistent framework.

2 Methodology

Cost of Financial Options and Guarantees (CFOG)

2.1 For products that provide guaranteed benefits, there is a cost to the insurance company and it's known as the cost of financial options and guarantees.

2.2 According to Principle 7 of the EEV Guidelines (refer to Appendix A), the scope of cost of the FOG is limited to the financial options and guarantees that impact policyholder benefits. It should be calculated based on stochastic techniques and should reflect the time value of the FOG.

2.3 In most situations, the cost of financial guarantees is defined as the time value of financial options and guarantees embedded in a product.

2.4 To value the cost, a stochastic model is more appropriate than a deterministic model as stochastic models enable us to value cases where the guarantees become effective when economic scenarios occur that might not be captured in the deterministic model.

2.5 In quantifying the cost, we need to perform both stochastic runs and the deterministic run. The difference in profit that arises from the two models indicates the cost to the company.

2.6 Stochastic runs are performed using different economic scenario indicators generated by the asset model while the deterministic run basically acts as the best estimate scenario. The best estimate scenario run is mathematically derived from the mean (or average) of all the stochastic economic scenarios.

Risk Neutral Valuation

2.7 Risk neutral valuation is a technique for valuing cash flows generated within a stochastic projection model, on a market consistent basis.

2.8 The characteristics of risk neutral valuation are as follows:

- It uses the risk-neutral probabilities when summing up all stochastic scenarios.
- It uses the risk-free rate as the deflator (or discount factor) when discounting cash flows
- The expected return for all asset types is always equal to the risk-free rate.

2.9 The deflator is associated only with the model and does not depend on the cash flow being value. This makes valuations of stochastic cash flows considerably simpler.

2.10 The main advantage of this valuation technique is to eliminate the need for a subjective choice of discount rate.

Detailed Formula of the Cost of Financial Options and Guarantees

2.11 To measure the cost of guarantees as at the valuation date in each stochastic scenario, we need to subtract the present value of distributable earnings occurring from the stochastic-projected cash flow from the present value of distributable earnings resulting from the deterministic projected cash flow.

2.12 Each stochastic scenario has different returns on assets and deflators for each projection period which were simulated from the economic scenario generator. Every stochastic run produced cash flows based on the asset returns and the distributable earnings are discounted using deflators.

2.13 As for the deterministic scenario, its economic parameters are the best estimate of all the stochastic economic scenarios. The best estimate on different asset returns as well as the deflators are taken as the mean (or average) of the different asset returns and deflators of all stochastic scenarios respectively. The cash flows projected are calculated together with the distributable earnings and discounted using its deterministic parameters.

2.14 Each different economic scenario shows a different cost to the company.

2.15 The cost of FOG is the average of the cost in all scenarios. We derive the following formula:

$$\frac{\sum_{i=1\,to\,n}\begin{pmatrix}\text{Present Value of Distributable Earnings} & - & \text{Present Value of Distributable Earnings}\\ \text{in Stochastic Scenario } i & & \text{in Deterministic Scenario}\end{pmatrix}}{n}$$

$$= \begin{matrix}\text{Average Present Value of Distributable Earnings} & - & \text{Present Value of Distributable Earnings}\\ \text{in Stochastic Scenario} & & \text{in Deterministic Scenario}\end{matrix}$$

2.16 The projection model is the combination of a liability model and asset model. More detailed explanation about each model is provided in the next section.

Asset modelling

2.17 Asset modelling supplies the returns on different asset types and deflators. These include return on equity, interest return on government bonds and risk-neutral deflators. Investment return with the 80:20 bond-equity are derived from equity and bond returns while crediting is defined as the maximum of 2% and the bond return less 1%. In addition a market interest rate assuming 50% equity and 50% bond has also been used.

2.18 Finally, deflators are used as the discount factor in discounting each period's distributable earnings of the product.

Liability modelling

2.19 A stochastic liability model which projects the following cash flow items for a single premium interest sensitive annuity product has been set up in the Watson Wyatt proprietary VIP system. The product features are listed in Appendix B. The projected cash flows are:

- Premium income
- Investment income
- Initial expenses
- Renewal expenses

- Commissions
- Death benefits
- Surrender benefits
- Maturity benefits
- Increase in reserves
- Stabilisation fund expenses
- Premium tax
- Corporate tax
- Cost of solvency margin
- Profit after tax and solvency margin / Distributable earnings

2.20 From the above cash flow items, cash flow items that are non-economic scenario sensitive such as premium income and commission paid show the same cash flows throughout all stochastic scenarios. Whereas cash flows items such as investment income and surrender outgo require input from the asset model to perform the calculations. These cash flows change from one stochastic scenario to the other. The bottom line of the cash flows, distributable earnings, is then discounted at the risk-neutral deflator to give us the present value of distributable earnings of the product.

Steps in calculating cost of the FOG excluding dynamic policyholders' interaction

2.21 This session lists out the steps in calculating the cost of the FOG excluding dynamic policyholders' interactions. The calculation can be summarised as the following:

Part I

- Run the liability model once using assumptions C1 (Deterministic column) listed in Appendix C.
- Calculate the present value of distributable earnings from the deterministic scenario using the risk-neutral deflator in assumptions C1.

Part II

- Perform 10,000 runs on the same model using assumptions C2 listed in Appendix C. (Stochastic-excluding dynamic interactions column)
- The economic assumptions for 10,000 scenarios are pre-generated from ESG (further explanation in Section 3)
- Calculate distributable earnings from each of the stochastic scenarios.
- The present value of distributable earnings in each scenario is calculated by discounting each scenario's distributable earnings at the scenario risk-neutral deflator.
- Finally, calculate the average deflated distributable earnings over the 10,000 scenarios.

Part III

- The cost of the FOG equals the average present value of distributable earnings from the stochastic scenarios less the present value of the distributable earning from the deterministic scenario
- Calculate the cost of the FOG as a percentage of the single premium

Steps in calculating cost of FOG including dynamic policyholders' interaction

2.22 This part discusses the considerations in calculating the cost of the FOG including dynamic policyholder interactions.

2.23 The best estimate discontinuance rate is normally derived from the historical industrial experience as well as past experience of an insurance company. Likewise, the dynamic discontinuance rate assumption can be transformed mathematically into a formula if there is credible and reliable experience segregated by different economic scenarios (bullish and bearish).

2.24 In addition to the use of past experience, there are several general principles that are sensible to follow when setting dynamic discontinuance rates:

- The dynamic discontinuance rates should be centred around the best estimate lapse rates. When a policy is projected under best estimate assumptions the dynamic discontinuance rates should be consistent with the best estimate lapse rate.
- The dynamic discontinuance rates should depend on a comparison between the return available via the insurance policy and an indication of the return available elsewhere in the market. For products with embedded options or guarantees, this is often achieved by basing the dynamic discontinuance rates on the degree of "in-the-moneyness" of the option and guarantee.
- It can be argued that the dynamic discontinuance rate should remain equal to the best estimate discontinuance rate for a central band of economic scenarios that present policyholders with a non-trivial reason to alter their discontinuance behaviour.
- At either side of this central band, the dynamic discontinuance should change in a manner that is advantageous to the policyholder, and thus disadvantageous to the life insurer.
- The discontinuance rates should be subjected to a maximum and a minimum.

2.25 Instead of having a deterministic discontinuance assumption as 3%, the suggested dynamic discontinuance assumptions were based on a floor at the minimum discontinuance rate, a cap at the maximum discontinuance rate and proportional discontinuance rates in between. The formula of the dynamic discontinuance for this model is set as below:

$$Discontinuance = \begin{cases} Max(1\%, 3\% \times x) & x < 80\% \\ 3\% & 80\% \le x \le 120\% \\ Min(20\%, 3\% \times x) & x > 120\% \end{cases}$$

$$where\ x = \frac{market\ rate}{crediting\ rate}$$

2.26 The exact same calculation in 2.21 is then performed except replace the assumption C2 used in step II (Stochastic runs) with assumptions C3 in Appendix C. This assumption set has incorporated the dynamic lapse assumptions.

3 Asset Modeling

Overview of Market Consitstent Asset Model

3.1 Market Consistent Asset Model is the Watson Wyatt in-house asset model that has been used in producing this report. This section discusses the overview of the asset model, the model calibration process and the underlying structure.

Model calibration process

3.2 Key requirements of a market consistent asset model are the ability to replicate today's market prices, simple calibration process, efficiency to support complex modelling exercise and transparency to aid understanding and demonstrate market consistency.

3.3 The model's underlying structure encloses theoretical mathematical models or closed-form formulae in calculations for different asset classes and purposes. The description of the mathematical model and their sources can be found in Appendix D.

3.4 The asset modelling processes are basically divided into three stages: data input, calibration and scenario generation.

3.5 The model is described by only a handful of parameters and variables that ease the calibration of the model to market data. The input to the model are the parameters that are derived directly from observable market prices (such as the yield curves and implied swaptions volatilities).

3.6 The purpose of swaption calibration is to derive the parameters that drive the risk free yield curve volatility. In the asset model, which makes use of the Hull-White model, the parameters are the short rate volatility and volatility reduction factor.

3.7 For the calibration of initial yield curves and risk-neutral deflator, input of the zero coupon curve is taken from Bloomberg and the term structure is fitted using the non-linear least squares technique.

3.8 As for calibration to the market prices of derivative instruments, swaptions are generally calibrated by extrapolating the volatility structure.

3.9 Scenario generation then produces different economic scenarios that are required for valuations such as equity total return and whole yield curve projections.

3.10 The output from the model consists of stochastic simulations of risk-neutral deflator, interest rates and equity return.

Deflator

3.11 The deflators are a central building block of the model and provide a mechanism to produce market-consistent results. The deflator approach provides a robust mechanism for valuing interest guarantees and other embedded options within life assurance policies in a way that is consistent with option pricing theory.

3.12 The market or "fair" value of the insurance liability (including any guarantees or options) will be the mean value of ($cashflow_{i,t}$ * $deflator_{i,t}$) for each scenario *i* and time *t*.

3.13 Risk-neutral deflator is chosen such that the initial yield curve can be replicated from future projections, i.e. the mean value of $deflator_{i,t}$ for each scenario i and time t shall be equal to the time zero t-year zero-coupon risk-free bond price (refer to 3.23)

Empirical Tests

3.14 Market consistency tests endorse the capability of the model in modelling fixed income instruments, equity and cash total returns and in pricing contingent claims such as swaptions and forward equity options.

3.15 We use different empirical tests for different asset classes. The tests include zero coupon bond tests, deflator tests and equity price tests.

3.16 The zero coupon bond tests indicate that the model exactly replicates the market prices of zeros (refer to 3.24).

3.17 Equity price tests perform checks on the level of equity and cash total return indices. Results show that the deflator methodology replicates the starting level of two indices with a marginal degree of error over 10,000 scenarios (refer to 3.25).

Asset Modelling in this report

Input

3.18 The asset model was calibrated using the following data:

- Taiwan market swap curve as at 30 June 2006
- Implied volatilities of at-the-money interest rate swaptions as at 30 June 2006
- Implied volatilities of at-the-money equity options as at 30 June 2006

3.19 We used swaptions with combinations of 1, 5 and 10 years for tenure and 1, 2, 3 and 5 years for maturity in the calibration. Each swaption carries an equal weight.

Output

3.20 An economic scenario file containing 10,000 simulations of risk-neutral fixed-income assets, equities has been applied. The scenario file was calibrated to the Taiwan market as at 30 June 2006.

3.21 The scenario file was generated using an in-house MCAM. The model produces arbitrage free economic scenarios containing one year return of fixed-income assets, total return of equities and risk neutral deflators which are consistent with the market prices of assets.

Empirical tests on market consistency

3.22 Market consistency tests were carried out on the scenario file. Ideally, when the simulated asset prices are discounted by the risk-neutral deflators the expected value should be close to the initial price of the asset, implying that in a risk-neutral world all assets are expected to earn a risk-free interest rate.

3.23 The table below compares the average of risk-neutral deflators (model) with the deflator implied by the initial yield curve (market).

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Model	98.23%	96.20%	93.98%	91.64%	89.24%	86.82%	84.39%	82.01%	79.68%	77.42%
Market	98.23%	96.21%	93.98%	91.66%	89.30%	86.89%	84.53%	82.14%	79.86%	77.66%

3.24 The table below compares the discounted value of fixed-income assets (model) with its initial market value (market).

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Model	1.0005	1.0006	1.0010	1.0010	1.0009	1.0004	1.0003	1.0003	1.0002	1.0003
Market	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

3.25 The table below compares the discounted value of equities (model) with its initial market value (market).

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Model	0.9999	0.9981	0.9996	0.9981	0.9984	0.9973	0.9971	0.9956	0.9960	0.9973
Market	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

3.26 The portfolio is assumed to have a 80:20 split between fixed-income assets and equities. The portfolio values must be market consistent if the two underlying assets are both market consistent.

4 Results and Conclusion

Deterministic Scenario Results

4.1 The deterministic scenario is also known as best estimate scenario. This is a deterministic scenario where the economic scenario parameter is the best estimate of the stochastic economic scenarios.

4.2 The detailed cash flows for the best estimate scenario determined by the methodology and assumptions specified are as follows:

Projection year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Premium Income	600,000	0	0	0	0	0	0	0	0	0
Investment Income	10,293	12,230	13,599	14,296	14,740	15,163	15,026	15,414	14,839	14,477
Expenses Outgoes	2,300	125	123	121	119	117	116	114	112	110
Commission Outgoes	15,840	0	0	0	0	0	0	0	0	0
Death Outgo	1,618	2,253	2,579	2,718	2,853	2,992	3,142	3,311	3,506	3,727
Surrender Outgo	17,338	17,189	17,113	17,027	16,978	16,836	16,564	16,291	16,015	15,736
Maturity Outgo	0	0	0	0	0	0	0	0	0	511,966
Increase in Reserve	592,031	(8,517)	(8,750)	(8,796)	(8,837)	(8,8810	(8,936)	(9,0100	(9,108)	(521,197)
Stab Fund Expenses	600	0	0	0	0	0	0	0	0	0
Premium Tax	176	(216)	(216)	(217)	(218)	(215)	(211)	(208)	(204)	(201)
Gross Profit	(19,610)	1,395	2,751	3,442	3,844	4,315	4,352	4,917	4,519	4,337
Corporate Tax	(6,373)	453	894	1,119	1,249	1,402	1,414	1,598	1,469	1,409
Net Profit	(13,237)	942	1,857	2,323	2,595	2,912	2,937	3,319	3,050	2,927
Cost Of Solvency Margin	35,522	(851)	(903)	(925)	(940)	(955)	(954)	(9700	(960)	(31,675)
Distributable Earnings	**(48,759)**	**1,793**	**2,760**	**3,248**	**3,535**	**3,867**	**3,892**	**4,288**	**4,010**	**34,602**

Table A: Detailed cash flow for best estimate scenario

4.3 In producing the above cash flows, the best estimate of bond return and investment return rate are used. The deflator is simply the reciprocal of zero-coupon government bond yield with different maturity years. The different rates used in the best estimate scenario are listed below:

Projection year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Crediting Rate	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%
Bond Return	1.80%	2.10%	2.37%	2.53%	2.65%	2.77%	2.79%	2.91%	2.85%	2.83%
Investment	1.80%	2.10%	2.37%	2.53%	2.65%	2.77%	2.79%	2.91%	2.85%	2.83%

Return										
Discontinuance Rate	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
Risk-Neutral Deflator	0.9823	0.9621	0.9398	0.9166	0.8930	0.8689	0.8453	0.8214	0.7986	0.7766

Table B: Different rates used in best estimate scenario

4.4 The crediting rate is the maximum of 2% or the bond return less 1%. As the bond return for all years in the deterministic scenario is constantly below 3%, the crediting rate remains constant at 2% annually.

4.5 The risk-neutral deflator decreases over the projection years and it's consistent with the time value of money, where the money as of today is always worth more than the same amount of money tomorrow.

4.6 It should be noted that the bond return and investment return for each projection period are exactly the same. This is consistent with the definition of risk-neutral valuation where the expected return (or best estimate) of different asset class should be equal to the risk-free rate.

4.7 The present value of distributable earnings in the best estimate scenario using the above deflator gives us TWD 2,805.

CFOG excluding policyholder's dynamic behaviour

4.8 A stochastic model is used to produce 10,000 projection cash flows for the 10,000 different economic scenarios.

4.9 The average cash flows for all the stochastic runs are calculated as following:

Projection year (Average CF)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Premium Income	600,000	0	0	0	0	0	0	0	0	0
Investment Income	10,488	12,279	14,146	14,619	15,567	15,746	16,336	16,348	16,433	16,568
Expenses Outgoes	2,300	125	123	121	119	117	116	114	112	110
Commission Outgoes	15,840	0	0	0	0	0	0	0	0	0
Death Outgoes	1,626	2,286	2,641	2,808	2,971	3,137	3,314	3,512	3,737	3,993
Surrender Outgoes	17,424	17,441	17,526	17,590	17,678	17,652	17,473	17,280	17,073	16,858
Maturity Outgoes	0	0	0	0	0	0	0	0	0	549,854
Increase in Reserve	597,823	(3,011)	(3,526)	(4,251)	(4,780)	(5,612)	(6,102)	(6,584)	(6,982)	(556,975)
Stab Fund Expenses	600	0	0	0	0	0	0	0	0	0
Premium Tax	60	(330)	(328)	(319)	(314)	(298)	(288)	(279)	(271)	(269)
Gross Profit	(25,184)	(4,232)	(2,289)	(1,330)	(107)	750	1,824	2,305	2,764	2,999

Corporate Tax	(8,185)	(1,375)	(744)	(432)	(35)	244	593	749	898	975
Net Profit	(16,999)	(2,857)	(1,545)	(898)	(72)	506	1,231	1,556	1,866	2,024
Cost Of Solvency Margin	35,869	(522)	(605)	(662)	(720)	(775)	(821)	(850)	(877)	(33,880)
Distributable Earnings	**(52,869)**	**(2,335)**	**(940)**	**(2360**	**648**	**1,281**	**2,052**	**2,406**	**2,742**	**35,904**

Table C: Detailed average cash flows for 10,000 stochastic scenario runs for case excluding policyholder's dynamic interactions

4.10 Different rates have been used in each of the stochastic scenario for the determination of CFOG excluding policyholder's interactions. An average of the rates generated from ESG and the discontinuance rate are presented as below:

Projection year (Average Rate)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Crediting Rate	3.00%	2.97%	2.94%	2.85%	2.78%	2.66%	2.60%	2.54%	2.50%	2.51%
Bond Return	1.85%	2.13%	2.40%	2.55%	2.70%	2.77%	2.89%	2.96%	2.99%	3.01%
Investment Return	1.83%	2.09%	2.43%	2.53%	2.72%	2.77%	2.90%	2.93%	2.98%	3.03%
discontinuance Rate	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
Risk-Neutral Deflator	0.9823	0.9620	0.9398	0.9164	0.8924	0.8682	0.8439	0.8201	0.7968	0.7742

Table D: Different average rates used in 10,000 stochastic scenario runs for case excluding policyholder's dynamic interactions

4.11 The crediting rate has been defined as the maximum of 2% or the bond return less 1%. We can see that average crediting rate is always well above 2% which is consistent with our expectation. However, there isn't a direct relationship between the crediting rate and bond return rate due to the distortion by the application of the 2% floor for the cases where bond return is lower than 3% as well as by the impact of decreasing volatility feature of the stochastic model across the projection years (the longer the outstanding duration to maturity, the higher the volatility of bond return).

4.12 We also observe that the average bond return in each projection period is very close to the average investment return in the same projection period. This is consistent with the definition of risk-neutral valuation where the expected return (or average) of different asset class should be equal to the risk-free rate.

4.13 By excluding the policyholder's dynamic interactions, we assume that the policyholder is indifferent towards surrender decision making in different market scenarios where a 3% discontinuance rate has been assumed in each of the stochastic scenarios.

4.14 The average present value of the distributable earnings using the risk-neutral deflator for the stochastic scenarios is TWD (20,411).)

4.15 The cost of the FOG is taken as the difference between the average present value of distributable earnings in the stochastic runs and the present value of distributable earnings in the deterministic run. Therefore, the cost of the FOG in this case is TWD 23,215, which is equivalent to 3.87% of the gross premium.

CFOG including policyholder's dynamic behaviour

4.16 The same stochastic model with some modifications to incorporate dynamic discontinuance rate, is used to produce 10,000 projection cash flows for the 10,000 different economic scenarios.

4.17 The average cash flows determined by the methodology and assumptions specified are as follows:

Projection year (Average CF)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Premium Income	600,000	0	0	0	0	0	0	0	0	0
Investment Income	10,526	12,589	14,808	15,620	16,970	17,502	18,510	18,869	19,326	19,860
Expenses Outgoes	2,300	129	129	130	131	131	131	132	132	132
Commission Outgoes	15,840	0	0	0	0	0	0	0	0	0
Death Outgoes	1,642	2,357	2,777	3,011	3,249	3,498	3,767	4,069	4,414	4,806
Surrender Outgoes	5,854	6,050	6,499	6,692	7,026	7,175	7,440	7,434	7,499	7,667
Maturity Outgoes	0	0	0	0	0	0	0	0	0	668,173
Increase in Reserve	610,096	9,317	8,587	7,861	7,172	6,251	5,485	4,952	4,411	(664,132)
Stab Fund Expenses	600	0	0	0	0	0	0	0	0	0
Premium Tax	(196)	(352)	(355)	(349)	(347)	(336)	(331)	(326)	(324)	(327)
Gross Profit	(25,609)	(4,911)	(2,829)	(1,725)	(2610	783	2,018	2,609	3,194	3,542
Corporate Tax	(8,323)	(1,596)	(919)	(561)	(85)	254	656	848	1,038	1,151
Net Profit	(17,286)	(3,315)	(1,910)	(1,165)	(176)	528	1,362	1,761	2,156	2,391
Cost Of Solvency Margin	36,606	211	106	40	(39)	(109)	(183)	(225)	(270)	(40,398)
Distributable Earnings	**(53,892)**	**(3,525)**	**(2,015)**	**(1,204)**	**(137)**	**638**	**1,545**	**1,986**	**2,426**	**42,789**

Table E: Detailed average cash flows for 10,000 stochastic scenario runs for case including policyholder's dynamic interactions

4.18 The average of different rates used in the stochastic scenario for the determination of CFOG incorporating policyholder's interactions:

Projection year (Average Rate)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Crediting Rate	3.00%	2.97%	2.94%	2.85%	2.78%	2.66%	2.60%	2.54%	2.50%	2.51%
Bond Return	1.85%	2.13%	2.40%	2.55%	2.70%	2.77%	2.89%	2.96%	2.99%	3.01%
Investment Return	1.83%	2.09%	2.43%	2.53%	2.72%	2.77%	2.90%	2.93%	2.98%	3.03%

Discontinuance Rate	1.01%	1.02%	1.07%	1.08%	1.10%	1.11%	1.14%	1.13%	1.13%	1.14%
Market Rate	1.81%	2.05%	2.47%	2.49%	2.74%	2.78%	2.91%	2.89%	2.97%	3.08%
Earn Credit Ratio	0.5368	0.6361	0.8438	0.8860	1.0033	1.0597	1.1327	1.1370	1.1729	1.2079
Risk-Neutral Deflator	0.9823	0.9620	0.9398	0.9164	0.8924	0.8682	0.8439	0.8201	0.7968	0.7742

Table F: Different average rates used in 10,000 stochastic scenario runs for case including policyholder's dynamic interactions

4.19 An additional economic scenario indicator, the market rate is applied in this model. The market rate represents the rate available in the market to policyholders if they choose not to continue the insurance contract. In this model, it is defined as the average of bond rate and equity rate (50:50 bond-equity).

4.20 The earn-credit ratio is defined as the expected additional financial benefit (measured as a percentage) that policyholders are able to gain by surrendering the current insurance contract and investing money in other potential investments in the market. It is calculated as the market rate divided by the crediting rate and is used in the determination of policyholders' discontinuance rate. From the table, we observe that the higher the ratio (i.e. the better the investment opportunity elsewhere in the market), the higher the discontinuance rate of policyholders. The exact formula of the discontinuance rate can be found in Section 2.25.

4.21 When we examine the dynamic discontinuance rate formulae, we expect to see the discontinuance rate to be equal or greater than 3% for cases where the market rate is higher than crediting rate. However, it does not seem to be the case when we look at the average numbers presented here. The reason is that the relationship mentioned has been distorted as the rates observed are actually average from 10,000 scenarios. The expected relationship will only be observed if we look at discontinuance rate versus earn-credit ratio on one-to-one basis in each respective scenario.

4.22 The average present value of the distributable earnings using the risk-neutral deflator for the stochastic scenarios is TWD (21,497).

4.23 The cost FOG is taken as the difference between the average present value of distributable earnings in the stochastic runs and the present value of distributable earnings in the deterministic run. Therefore, the cost of the FOG in this case is TWD 24,301 which is equivalent to 4.05% of the gross premium.

Comparison of results for CFOG excluding policyholder's behaviour and including policyholder's behaviour

4.24 The only modelling difference between the cost of the FOG numbers in 4.15 and 4.23 is the inclusion of dynamic behaviour from policyholder in the latter case.

4.25 The two tables below (Table G and H) are identical with Table D and F above.

4.26 Table G summarises the average of a few indicators including the crediting rate, bond return, investment return, annual discontinuance rate and risk-neutral deflator by policy year for the model which excludes policyholder's dynamic behaviour. Table H summarises the average rates for the model that includes policyholder's dynamic behaviour with two extra indicators: market rates and earn-credit ratio.

Projection year (Average Rate)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Crediting Rate	3.00%	2.97%	2.94%	2.85%	2.78%	2.66%	2.60%	2.54%	2.50%	2.51%
Bond Return	1.85%	2.13%	2.40%	2.55%	2.70%	2.77%	2.89%	2.96%	2.99%	3.01%
Investment Return	1.83%	2.09%	2.43%	2.53%	2.72%	2.77%	2.90%	2.93%	2.98%	3.03%
Discontinuance Rate	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
Risk-Neutral Deflator	0.9823	0.9620	0.9398	0.9164	0.8924	0.8682	0.8439	0.8201	0.7968	0.7742

Table G: Average crediting rate, bond return, investment return, annual discontinuance rate and risk-neutral deflator in 10,000 scenarios for case which excludes policyholder's dynamic behaviour.

Projection year (Average Rate)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Crediting Rate	3.00%	2.97%	2.94%	2.85%	2.78%	2.66%	2.60%	2.54%	2.50%	2.51%
Bond Return	1.85%	2.13%	2.40%	2.55%	2.70%	2.77%	2.89%	2.96%	2.99%	3.01%
Investment Return	1.83%	2.09%	2.43%	2.53%	2.72%	2.77%	2.90%	2.93%	2.98%	3.03%
Discontinuance Rate	1.01%	1.02%	1.07%	1.08%	1.10%	1.11%	1.14%	1.13%	1.13%	1.14%
Market Rate	1.81%	2.05%	2.47%	2.49%	2.74%	2.78%	2.91%	2.89%	2.97%	3.08%
Earn Credit Ratio	0.5368	0.6361	0.8438	0.8860	1.0033	1.0597	1.1327	1.1370	1.1729	1.2079
Risk-Neutral Deflator	0.9823	0.9620	0.9398	0.9164	0.8924	0.8682	0.8439	0.8201	0.7968	0.7742

Table H: Average crediting rate, bond return, investment return, annual discontinuance rate, market rate, earn-credit ratio and risk-neutral deflator in 10,000 scenarios for case which includes policyholder's dynamic behaviour

4.27 It should be noted that this product is assumed to be supported by a portfolio that consists of 80% government bonds and 20% equities. The investment return is therefore the sum of 80% of the annual government bond return and 20% of the equity return.

4.28 For the case where policyholder's dynamic behaviour is incorporated, we have assumed that the market rate available to the policyholder elsewhere is a combination of a 50:50 bond-equity portfolio and it has taken the average of government bond returns and equity returns.

4.29 The market to crediting rate ratio (earn-credit ratio) is calculated as the market rate divided by the product crediting rate. The ratio is then used to calculate the dynamic discontinuance rate of policyholder.

4.30 This product's crediting rate is defined as the bond return less 1%, subject to a minimum of 2% per annum.

4.31 As both tables are generated from the similar 10,000 stochastic economic scenarios, we observe that the average of the crediting rates, bond returns, investment returns and risk-neutral deflators are exactly the same.

Observations

4.32 A few observations can be drawn from the tables. The average crediting rate for all years is always greater than 2% due to the guaranteed rate of the 2% floor provided in the product. There is no clear correlation between the average crediting rates and average bond return although the crediting rate is defined as a function of the bond return. This is because the correlation has been distorted by the application of the 2% floor for the cases where bond return is lower than 3% as well as by the impact of decreasing volatility feature of the stochastic model across the projection years (the longer the outstanding duration to maturity, the higher the volatility of bond return).

4.33 In projection year 1 to 5, the average investment return has been constantly lower than the average bond return. This has led to the very negative distributable earnings for the mentioned years.

4.34 Table G demonstrates that the policyholder is indifferent in making the surrender decision under different market scenarios. On the other hand, with the inclusion of policyholder's dynamic behaviour modelling, Table H shows an apparent positive correlation between the annual discontinuance rate and earn-credit rate ratio. This means a higher discontinuance rate is observed when the market rate is comparably higher than the product crediting rate. This is consistent with what we expect as the policyholder will tend to maintain the contract when there is no better value investment opportunity in the market.

Results and Conclusion

4.35 From 4.15, we can see that the cost of the FOG accounts for 3.87% of the single premium for this product before considering the policyholder's dynamic behaviour. In terms of pricing, it means that the cost of offering the guaranteed features in this product to the company is 3.87% of the single premium. Ideally, this cost needs to be properly reflected in the pricing by factoring the percentage into the premium rates.

4.36 Further investigation in the following sections shows that by using the model which incorporates policyholder behaviour, cost of the FOG has been increased to 4.05% of the single premium which is a total increase of 5% extra cost to the company.

4.37 The management of the insurance company should be aware that the incorporation of dynamic policyholder behaviour does have an impact on the results. In order to model interactions, the company ought to have credible past years experience on its policyholder reaction and continual observation on the change of behaviour throughout all years. The company should make sure that with more complicated modeling, it can better reflect reality and hence be able to measure the cost of the FOG more accurately.

4.38 For companies with no credible experience available, we suggest that the companies should keep the model simple to avoid complications in the model.

5 Limitations and Suggestions

Limitations

5.1 This report focuses on this particular single premium interest sensitive product with product features as given in Appendix A. In addition, it only examines the case of male, 45. No runs nor analysis have been performed on other products or similar products with different model points. Therefore, the conclusions drawn in this report are only valid for this specific product with this particular model point.

5.2 Assumptions have been made about future economic conditions as at 30 June 2006. These assumptions have been made on the basis of reasonable estimates and with reference to general actuarial assumptions in the Taiwan insurance industry. However, actual future experience is likely to differ from these assumptions, due to random fluctuations, changes in the operating environment and other factors. Such variations in experience could have a significant effect on the results and conclusions.

5.3 The scenario file has been generated using stochastic techniques and is therefore subject to statistical error, the impact of which generally reduces as the number of scenarios increases. 10,000 scenarios were used. The use of a different number of scenarios, or a different set of scenarios from the same asset model may lead to different results.

Suggestions

5.4 This report only considers policyholder's dynamic behaviour in different economic scenarios.

5.5 The interactions involved in reality is not only limited to the policyholder's dynamic interactions. Some other examples of interactions include policy loan take-up rates which depend on the interest rates from the economic model and management interactions such as asset purchases and sales based on projected cashflows from the liabilities, asset allocation depending on the surplus of the company and bonus declarations based on the projected level of surplus and current yields are some other interaction examples. These interactions should be considered in the actuarial modelling, depending on how important the impact is to the company.

Appendix A – EEV Principles and Guidelines

Principle 1: Embedded Value (EV) is a measure of the consolidated value of shareholder's interests in the covered business.

Principle 2: The business covered by the EVM should be clearly identified and disclosed.

Principle 3: EV is the present value of the shareholder's interests in the earnings distributable from assets allocated to the covered business after sufficient allowance for the aggregate risks in the covered business. The EV consists of the following components:

- free surplus allocated to the covered business
- required capital, less the cost of holding required asset
- present value of future shareholder cash flows from in-force covered business (PVIF)

Principle 4: The free surplus is the market value of any capital and surplus allocated to, but not required to support, the in-force covered business at the valuation date.

Principle 5: Required capital should include any amount of assets attributed to covered business over and above that required to back liabilities for covered business whose distribution to shareholders is restricted. The EV should allow for the cost of holding the required capital.

Principle 6: The value of future cash-flows from in-force covered business is the present value of future shareholder cash flows projected to emerge from the assets backing liabilities of the in-force covered business ("PVIF"). This value is reduced by the value of financial options and guarantees as defined in Principle 7.

Principle 7: Allowance must be made in the EV for the potential impact on future shareholder cash flows of all financial options and guarantees within the in-force covered business. This allowance must include the time value of financial options and guarantees based on stochastic techniques consistent with the methodology and assumptions used in the underlying embedded value.

Principle 8: New business is defined as that arising from the sale of new contracts during the reporting period. The value of new business includes the value of expected renewals on those new contracts and expected future contractual alterations to those new contracts. The EV should only reflect in-force business, which excludes future new business.

Principle 9: The assessment of appropriate assumptions for future experience should have regard to past, current and expected future experience and to any other relevant data. Changes in future experience should be allowed for in the value of in-force when sufficient evidence exists and changes are reasonably certain. The assumptions should be actively reviewed.

Principle 10: Economic assumptions must be internally consistent and should be consistent with observable, reliable market data. No smoothing of market or account balance values, unrealised gains or investment return is permitted.

Principle 11: For participating business the method must make assumptions about future bonus rates and the determination of profit allocation between policyholders and shareholders. These assumptions should be made on a basis consistent with the projection assumptions, established company practice and local market practice.

Principle 12: Embedded value results should be disclosed at consolidated group level using a business classification consistent with the primary statements.

Appendix B – Product feature

1 Insured / Policyholder

- male 45, non-smoker

2 Currency

- TWD

3 Policy Term

- 10 years

4 Premium Type

- Single Premium

5 Front-end charge

  - 0% of single premium

6 Guaranteed benefits

- Death benefit – 100% of the account value is payable upon death during the policy term
- Surrender value – Account value after deduction of surrender penalty
- Surrender penalty
  - 4.5% year 1
  - 3.5% year 2
  - 2.5% year 3
  - 1.5% year 4
  - 0% year 5onwards

7 Crediting rate is maximum of 2% and government bond yield rate after 1% deduction

8 No dividends are payable

Appendix C – Assumptions

Assumptions for deterministic run (C1), stochastic runs excluding policyholder's dynamic behaviour (C2) and stochastic runs excluding policyholder's dynamic behaviour (C3)

	Deterministic scenario (C1)	**Stochastic scenarios – Excludes interactions (C2)**	**Stochastic scenarios – Includes interactions (C3)**
Premium size	Single Premium NTD 600,000	Single Premium NTD 600,000	Same as C2
Mortality experience	TSO 89 with selection factor of 50% in year 1 65% in year 2 70% ultimate	TSO 89 with selection factor of 50% in year 1 65% in year 2 70% ultimate	Same as C2
Discontinuance rates	3% per year	3% per year	**max (1%, 3% * x)** for x < 80% **3%** for 80% <= x <= 120% **min (20%, 3% * x)** for x > 120% where x = market rate/ crediting rate
Expenses	Acquisition NTD2300, plus Renewal NTD127.50 per policy Inflation at 2% per annum	Acquisition NTD2300, plus Renewal NTD127.50 per policy Inflation at 2% per annum	Same as C2
Commission	2% year 1 0% thereafter	2% year 1 0% thereafter	Same as C2
Overriding Commission	32% year 1 0% thereafter	32% year 1 0% thereafter	Same as C2
Investment Return	Risk-Free rate (with zero volatility)	Risk-Free rate (with 80:20 equity-bond volatility)	Same as C2
Market Rate	N/A	N/A	Risk-Free rate (with 50:50 equity-bond volatility)
Stabilisation Fund	0.1% of premium income	0.1% of premium income	Same as C2
Premium Tax	2% of (premium income + surrender gains – release in reserve)	2% of (premium income + surrender gains – release in reserve)	Same as C2
Corporate Tax	32.5% of pre-tax income	32.5% of pre-tax income	Same as C2

Risk discount rate	Deflator implied by the initial yield curve (market)	Risk neutral scenario deflator (with zero volatility)	Same as C2
Statutory reserving method	Account value	Account value	Same as C2
Solvency margin	6% of statutory reserve	6% of statutory reserve	Same as C2
Payment mode	Single Premium	Single Premium	Same as C2
Projection period	Up to benefit term	Up to benefit term	Same as C2

Appendix D – ESG Model Description

Risk-neutral deflator

Risk-neutral deflator at time t is defined as:

$$D_t = \frac{1}{CBI_t}$$

Where: CBI_t: cash bond index at time t

A cash bond is accumulated over time by the short rates, i.e., the instantaneous short rates.

The short rate model used is a one-factor Hull-White model. The details of this model are specified in the following papers:

- John Hull and Alan White, "Using Hull-White interest rate trees," Journal of Derivatives, Vol. 3, No. 3 (Spring 1996), pp 26-36
- John Hull and Alan White, "Numerical procedures for implementing term structure models I," Journal of Derivatives, Fall 1994, pp 7-16
- John Hull and Alan White, "Numerical procedures for implementing term structure models II," Journal of Derivatives, Winter 1994, pp 37-48
- John Hull and Alan White, "The pricing of options on interest rate caps and floors using the Hull-White model" in Advanced Strategies in Financial Risk Management, Chapter 4, pp 59-67.
- John Hull and Alan White, "One factor interest rate models and the valuation of interest rate derivative securities," Journal of Financial and Quantitative Analysis, Vol 28, No 2, (June 1993) pp 235-254
- John Hull and Alan White, "Pricing interest-rate derivative securities", The Review of Financial Studies, Vol 3, No. 4 (1990) pp. 573-592

Government Zero Coupon Bonds

In a risk-neutral set-up, the one-factor Hull-White model also defines the fixed-income asset prices over time. Government zero coupon bond has been the fixed-income asset used in this project.

The calibration method of the one-factor Hull-White model to the market data is specified in the following paper:

- MARC HENRARD, " EXPLICIT BOND OPTION AND SWAPTION FORMULA IN HEATH-JARROW-MORTON ONE FACTOR MODEL" Derivatives Group, Banking Department, Bank for International Settlements, CH-4002 Basel (Switzerland)

Equity Model

The equity model used is a geometric Brownian motion model, which is used by Black and Scholes as a risk-neutral option pricing framework.

- Black, Fischer, Myron Scholes (1973). "The Pricing of Options and Corporate Liabilities". Journal of Political Economy 81 (3): 637-654.
- Merton, Robert C. (1973). "Theory of Rational Option Pricing". Bell Journal of Economics and Management Science 4 (1): 141-183.