Pricing variable annuity product in Hong Kong -- a starting point for practitioners

Abstract

With regard to its rapid emergence in Hong Kong, this paper aims to perform a pricing exercise for a sample variable annuity (VA) product, using pricing considerations and assumptions that are specific to the Hong Kong market. The paper starts with an appreciation of how a typical VA product works in principle as well as a differentiation with similar types of insurance products such as unit-linked, both from a marketing perspective and a risk perspective. A stochastic pricing approach has been adopted, with particular considerations to assessing the cost of various types of death and living benefit guarantees that are commonly offered along with a VA product. Furthermore, a significant part has been devoted to the construction of an economic scenario generator, as it serves as a key of how the cost of guarantees can be assessed. Finally, overall profitability is outlined to determine whether a right balance can be sought with respect to the entire product.

Introduction

In highly developed insurance markets such us the US and Japan, variable annuity (VA) has become increasingly popular during the 1990's. Their sales began to catch up to and even eclipsed unit-linked and fixed annuity sales. In order to differentiate VA products from their other close competitors and to attract a larger share of the insurance market, VA writers began adding various guarantee features which were entirely backed by the insurer on top of the policyholders' separate accounts. Examples included various types of guaranteed minimum death benefit (GMDB), guaranteed minimum withdrawal benefit (GMWB), and guaranteed minimum annuity benefit (GMAB). These features then became a significant part of VA marketing and, as competition became fiercer, VA writers were forced to offer more and more generous guarantees. The cost implications from these guarantee features have then become a huge challenge for insurers to manage.

This type of VA products with guarantees did not come into attention in the Hong Kong market until recently. Fueled by the continued rise of the stock market, together with the flattening of growth in traditional product sales, insurers in Hong Kong have started to look at VA as a way to further expand their market share. Additional living and death benefit guarantees, would serve as a differentiation to other investment products and mutual funds. As a result, VA with various guarantee features is believed to be a main focus for the Hong Kong market in the next few years.

This paper aims to perform a stochastic pricing study of a VA product, with particular emphasis on assessing the cost of guarantees for different types of death and living benefit guarantee designs. With pricing considerations and assumptions specific to the HK market, this paper serves to act as a starting point for pricing such a product in Hong Kong. Specifically, the purpose of this study includes:

- Understand how a variable annuity product works as well as its difference compared to a unit-linked product, both from a marketing perspective and a risk perspective;
- Construct an economic scenario generator (ESG) for stochastic pricing purpose, making use of regime-switching lognormal distribution and Monte-Carlo simulation;
- Assess the profitability of a VA product, based on a product design which referenced that of a common product in the US market;
- Assess the cost implications for offering different types of death and living benefit guarantee features, in addition to the basic VA product.

The next section of this report, Product design & description, gave details on specific product designs and guarantee features that were commonly found in other markets. A design was then suggested to be used as the basis for pricing in the rest of the report. Then, the Methodology & Assumptions section outlined the methodology of the entire pricing process and gave details on the projection assumptions used for profit testing purpose.

In the section that follows, Generation of Investment Scenarios, specific techniques for generating stochastic investment assumptions were outlined. Since these scenarios would then serve as a key assumption for pricing the VA product and its guarantees effectively, much effort has been put on generating the economic return scenarios, and enhancements compared to previous commonly used ESGs have also been emphasized. After that, the Results, Analysis & Discussion section, gave details on the profit testing results and analyzed the cost-benefit relationship of different types of guarantees. Finally, the Conclusion section concluded the whole pricing process and gave recommendations as to how the process can be enhanced or improved.

Product design and description

Here we refer a variable annuity product, same as how it is commonly defined, only to the accumulation phase of separate account funds, from issue (date when the separate account is setup) to maturity (end of accumulation phase). In the US, however, the maturity of these funds would often be associated with a conversion to a life annuity, which is referred to as the annuitization phase. For the sake of this report, we have only considered the accumulation phase, since pricing is always performed independently for these two phases despite of their association. In addition, it should be stressed that the stochastic technique employed in this project would only be relevant to pricing the accumulation phase, since traditional deterministic approach is always perceived to be sufficient in pricing the payout phase.

A basic VA product, in itself, is very similar to products such as mutual funds and unit trusts in terms of its investment nature. Historically, insurance companies have tried to differentiate VA products from these competitors, essentially by the following ways:

- Product charges and loadings: VA products are often offered with low front-end loads but higher renewal charges, in contrast to the high initial charges that are commonly found in mutual funds and unit trusts. This type of charging structure would be more attractive to people targeting shorter investment horizons, since few charges will have been deducted in case of early withdrawal.

- Fund choice selection: VA products are often offered with a wide selection of funds, as well as an ability to invest into different funds at the same time. Charges for switching between funds are often made minimal. This arrangement would be attractive to people who want to diversify their risk through investing into different funds but do not have the enough volume to do so.

- Additional guarantee features: this is perhaps the most important part which differentiates a VA from its competitors. A death benefit guarantee serves to link a policyholder's life insurance needs to its investment; and a withdrawal benefit guarantee serves to provide downside investment protection while retaining the upside potential. Furthermore, in contrast to unit-linked products which cover the cost of offering these guarantees via an explicit scale of cost of insurance (COI) charge, the charges for these guarantees are often embedded into the annual fund management charge (usually expressed as a % of account balance) for a VA product. This would be attractive since it appears that the guarantee is offered at "no" extra charge to policyholders.

Having provided the advantages of VA over its competitors, however, the main downside of offering a VA product is the difficulty in assessing the cost of guarantees to be offered, especially in the early pricing stage. The wide selection of funds often complicates the task further since investment performance, which is closely related to the cost of guarantee, would become harder to predict. Furthermore, since the guarantee charge is often embedded into the annual management fee (does not match with the increasing nature of the guarantee cost), mispricing can be more dangerous compared to a common unit-linked product.

Proposed Product Design

The product design that is used for pricing in the rest of the report is described as follows, which renders a simplified version of what is commonly found in the US market. For the fund choice selection, 8 funds have been included for the sake of this project. While more funds may have been provided for a product in the real marketplace, we believe that these 8 funds already represent a wide enough spectrum of differing risks and returns in order to allow meaningful comparison (details on simulating returns of each on these funds are given in section Generation of Investment Scenarios).

General Product Design	Benefit period of 30 years Single premium
Charges and loadings (Before guarantees)	2% of single premium charge HKD 28 of fixed policy fee per month 10 basis points of account balance per month Annual fund management spread of 1%, 0.5% & 0.2% for stock, bond & cash funds respectively
Fund selection	Cash or cash equivalent Hong Kong stock fund US stock fund UK stock fund Japan stock fund Global trading bond fund Global fixed income fund Emerging market bond fund

Finally, a selection of various death and withdrawal benefit guarantee features is included in our pricing. Note that in reality selecting different guarantee features would require different charges since each of these guarantees render different cost implications to insurer. In this project we have kept the guarantee charging structure constant for different guarantees chosen. At the end recommendations will be given as to which of these guarantees are affordable by this charging structure and which are not.

Also note that we have limited our test to only death and withdrawal guarantees (i.e. no other guarantee features such as annuitization guarantees). In addition, the following types of guarantee features which we included in our product are all commonly available in a conventional US VA product.

Guaranteed minimum death benefit (GMDB)		Return of net considerations guarantee
		Net considerations rolling up at 2%
		Net considerations rolling up at 4%
		Monthly ratchet guarantee
		2-year ratchet guarantee
		3-year ratchet guarantee
Guaranteed minimum withdrawal benefit (GMWB)		Return of premiums paid guarantee
		Net considerations rolling up at 2%
		Net considerations rolling up at 4%
		Monthly ratchet guarantee
		2-year ratchet guarantee
		3-year ratchet guarantee
Waiting period until guarantees become effective		5 years for GMDB
		8 years for GMAB
Guarantee charge	GMDB	No charge
	GMWB	8 basis points of AV per month

Note that a ratchet feature refers to guaranteeing the benefit to be at least the higher of last period's guarantee and last period's account value. Therefore, a monthly ratchet resets the guarantee level by month while a 5-year ratchet resets the guarantee level every 5 years. All the other guarantee features are straightforward.

Methodology and assumptions

Review of traditional actuarial literature: deterministic pricing

Traditional insurance product development has made use of deterministic pricing for many years. Though being widely accepted for many product types, it may not be so desirable for pricing a variable annuity product with guarantees:

- It fails to quantify the cost of guarantees since it makes use of only one "best estimate" investment scenario to calculate the results;
- It fails to recognize the effect of volatility on investment income;
- It fails to take care of interactions of policyholder behavior (e.g. dynamic lapse, mortality anti-selection).

Adopted approach: stochastic pricing

Stochastic pricing approach has been adopted for this exercise, in which a policy is run on a large number of scenarios before combining the various runs to obtain the aggregate result. Each scenario represents an equal probability of occurrence in the future time periods. Since the future investment returns are stochastically projected, this approach is able to quantify the cost of guarantees offered in a product. Future investment income can also be projected more accurately, as it can handle the investment for a combination of funds with different risk/return profiles. Obviously, this approach can also handle items such as dynamic lapse and mortality anti-selection.

For this exercise, two assumptions have been stochastically projected – investment return and lapse rate. Other assumptions, such as mortality and expense, have remained deterministic. The reason is that there is no clear evidence to support that these assumptions would behave in a stochastic manner, especially in the Hong Kong market. For example, there is not enough evidence that mortality would be dependent on both lapse or investment return, and theoretical phenomena such as mortality anti-selection may not be relevant as well. Another reason is that even if any of these other assumptions is really stochastic in nature, there is also no established method (or not enough credible data) to stochastically develop the assumption.

Each of the major assumptions is discussed as follows:

Investment return: a number of 500 investment scenarios have been produced and each of them represents an equal probability of occurrence in the future. For each investment scenario, the output includes the monthly return for a projection of 30 years (i.e. maximum length of benefit period), for each of the 8 investment funds offered. Therefore, there would be 2,880 monthly returns included in one scenario (i.e. 360 months times 8 funds). This scenario generation process involves the construction of an economic scenario generator (ESG) and performing Monte-Carlo simulation. Complete details of how the investment scenarios were generated are given in the next section Generation of Investment Scenarios.

Lapse rate: theoretically a dynamic lapse rate function would depend on historical investment performance as well as the current level of surrender charge. Since our product design does not have any surrender charge, the lapse rate would only depend on investment performance.

The following formula has been adopted:
= base rate + [benchmark return – (annualized return for last 3 years, adjusted for guarantee)] * dependency strength

- Base rate represents our best estimate lapse rate should the historical return equals the benchmark return at all times. It is a schedule which varies by policy year. This schedule should be largely consistent with the lapse experience of a standard unit-linked product for large-scale insurance company in Hong Kong.
  15% for 1st to 7th year, 12% for 8th year to 9th year, 10% for 10th year onwards
- Benchmark return represents the return in which policyholders would normally expect in the long-term. It varies by the fund selected:

Cash or cash equivalent	4%
Hong Kong stock fund	10%
US stock fund	10%
UK stock fund	10%
Japan stock fund	10%
Global trading bond fund	6%
Global fixed income fund	6%
Emerging market bond fund	7%

- Both benchmark return and annualized actual return for the last 3 years are based on a weighted average of returns for various funds.
- The actual annualized return for the last 3 years is adjusted by the presence of any guarantee elected, in order to reflect the "true" investment return which an investor would look at when making the "whether to lapse" decision. For example, in the presence of a return of net considerations guarantee, the actual return would be floored at 0% in any case.

- Dependency strength represents the degree to which investment return can affect the level of surrender. Before the waiting period, it is set to a very low strength 0.1 for all funds, since investors generally would wait until end of the waiting period in order to invoke the guarantee when investment performance is poor. After the waiting period, it is set to 0.3 for stock funds, 0.2 for bond funds, and 0.15 for cash funds, reflecting the fact that policyholders are usually more sensitive when investing in stock funds.
- The dynamic component is assumed to be effective starting from the 4th year, reflecting the fact that investors generally would have a degree of tolerance within the first few years even if investor performance is poor.

Mortality rate: it is currently set to a deterministic 75% of the HKAL93 table for all policy years. We have not included any selection factors since variable annuity products are usually sold on a simplified underwriting (or even no underwriting) basis. The level of mortality assumption is also consistent with industry experience, perhaps slightly towards the conservative end.

Commissions & Expenses: no in-depth investigation has been carried out for the setting of these assumptions. They are entirely based on our intuitive knowledge of the Hong Kong market instead of experience.

Commission: 50% for 1st year, 10% for 2nd to 5th year, 8% afterwards

Acquisition Expense: 16% of premium for 1st year

Renewal Expense: HKD 96 per year and 1.6% of premiums for all years

Profit measure: gross profit margin is used as the basis for assessing the profitability of policies. In addition, as stochastic approach has been adopted for this exercise, we have taken the mean across all scenarios:

Gross Profit Margin = Average [PV (gross profits)] / Average [PV (premiums paid)]

The annual investment earned rate is used as the discount rate for calculating the present values, which is based on a month-on-month forward rate basis. Since the portfolio can be invested into various funds with different investment performance, weighted average monthly returns are calculated for the discounting purpose.

Reserving basis: currently 110% of the account balance is held as the reserve, for which we have assumed the 100% of AV would represent the reserve for the basic variable annuity benefit, while the extra 10% of AV would represent the reserve for all its guarantees offered. The determination of this 10% is, obviously, not scientific, and may be even quite arbitrary in some sense. However, we believe that reserving is not the main focus of this project and can therefore be “approximated”. At the same time, since our profits are discounted at the policy’s investment earned rate, theoretically the effect of extra reserving on profit margin will be zero.

Modal point selection: we have only profit tested on one single modal point throughout the entire project – a policy issued for a 30-year old male with a benefit period of 30 years and premium size of HKD 100,000. This selection of only one modal point was mainly due to runtime concerns, and also partly due to the fact that the profitability of the basic product may not be so sensitive to policy characteristics such as issue age and sex. For the guarantees offered, GMWB is obviously not sensitive to these characteristics; while GMDB does (where the guarantee cost increases with age), we have ignored it for now and have tested the affordability only with respect to this modal point (i.e. no charge for GMDB offered to a 30-year old male).

This particular modal point chosen represents the best estimate for the average business mix of our anticipated portfolio. Average issue age is expected to be at around year 30, and the investment horizon would probably be until retirement ages (around age 60). The premium size also represents our expected average, perhaps slightly to the conservative end.

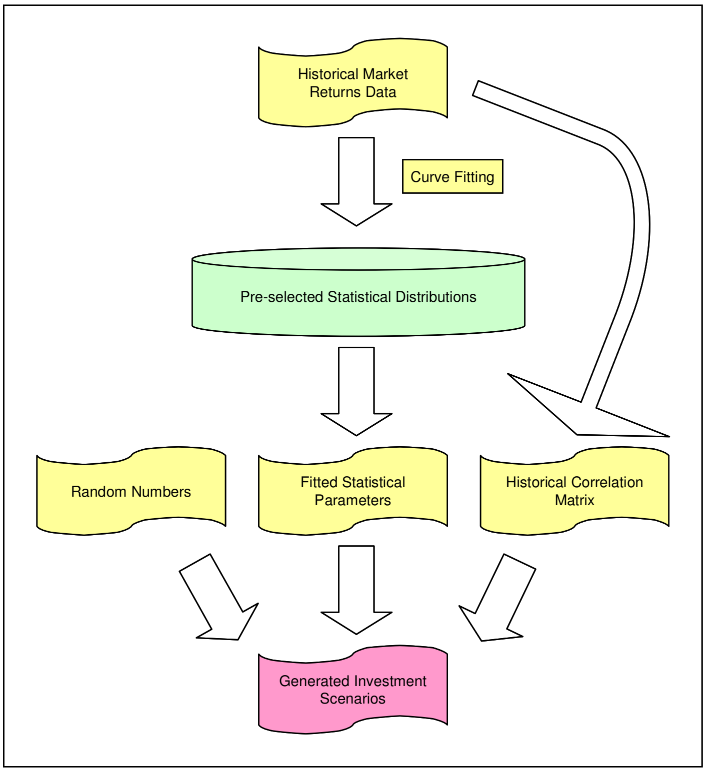
Finally, please note that the following has not been considered during this profit testing exercise:

- Effect of mortality anti-selection: in theory this should be catered especially when GMDB is being offered along with the product, since the presence of an in-the-money GMDB option would easily create incentive for more unhealthy people to join the policy / stay in the policy compared to healthy people (the latter is also known as selective lapsation).
- Tax: we have assumed our pricing exercise is done on a pre-tax basis.
- Reinsurance: we do not expect this would have any material impact on profitability.
- Cost of capital: we have assumed our pricing exercise is done on a pre-cost of capital basis.

Generation of investment scenarios

This section focuses on the detailed process regarding the setting of the stochastic investment assumptions, in which a large number of scenarios are generated using historical market data and statistical simulation techniques. Since investment scenarios are of primary importance towards assessing the cost of various guarantees, a detailed discussion of this scenario generation process has been separated into this independent section.

The entire investment scenario generation process can be summarized by the following diagram:



Before going in-depth into the detailed process, I would like to emphasize on some of the enhancements of this process compared to previous commonly used ESGs:

- The risk-free and risk premium components of the entire equity return have been modeled separately, in contrast to other common ESGs than they are being modeled one combined distribution. This is to reflect the independent nature between interest rate market and equity market;
- Returns for different market funds have been included with their correlations being modeled;
- Quasi-random numbers have been used for simulation purpose instead of pseudo-random numbers, allowing the set of numbers generated to be more robust.

(1) Collection of market returns data

Collect historical monthly return data from various market indices, in order to be used as the source for projecting future investment returns. In particular, one specific stock/bond/cash market index has been selected to represent each of the 8 funds to be offered in our product. The list of indices is as follows:

Fund offered in our product	Benchmark market index
Cash or cash equivalent	Hong Kong 3-month Treasury Yield Rate
Hong Kong stock fund	Hong Kong Hang Seng Index
US stock fund	US S&P 500 Index
UK stock fund	UK FTSE 100 Index
Japan stock fund	Japan Nikkei 225 Index
Global trading bond fund	CGBI WGBI World Bond Total Return Index
Global fixed income fund	Solomon Smith Barney World Bond Index
Emerging market bond fund	JP Morgan Global Bond Total Return Index

In addition, the following points need to noted:

- 20 years of monthly data have been collected for the stock & cash returns (the first 5 funds), while only 10 years of monthly data have been collected for the bond returns. Ideally 20 years of data for all indices would be more desirable; however, historical data for the bond funds were not easily available for the period longer than 10 years.
- Historical monthly dividend yields have been included in the stock index return, which means that the return data actually represent return from stock price appreciation plus dividend yield. The dividend yields of these stock indices, in contrast to individual stocks, came from weighted averages of reported dividend yields of the constituent stocks.

(2) Fitting of data into statistical distributions

In this stage, each of the historical index return data need to be fitted into a statistical distribution, such that the resulting parameters can be used for projecting future returns. As a first step, mean and volatility of these historical returns were calculated as a verification of the reasonableness of data:

	Treasury Short Rate	Hang Seng Index	S&P 500 Index	FTSE 100 Index	Nikkei 225 Index	EM Bond	Global Bond	SSB Bond
Average of LN of monthly returns	-0.04%	1.09%	0.88%	0.72%	0.19%	0.84%	0.43%	0.41%
Average volatility of LN of monthly returns	6.12%	8.30%	4.27%	4.59%	6.18%	4.25%	1.94%	0.45%
Average of LN of monthly risk premium	N/A	0.73%	0.51%	0.36%	-0.18%	0.56%	0.14%	0.12%
Average volatility of LN of monthly risk premiun	N/A	8.31%	4.27%	4.59%	6.18%	4.25%	1.97%	0.45%

In the above, mean and volatility of both the monthly returns and monthly risk premiums (defined as return minus treasury short rate) have been calculated. These would be useful for the next step of curve fitting process, as the treasury risk-free return and the various risk premiums for different stock indices would be fitted separately. A few points are worthwhile to note:

- The Japan market appears to be inferior to investing in other stock markets (lower mean but higher volatility), and has an even lower average return than the treasury rate. We suspect that the length of the data collection period has had a huge impact on these results, especially when Japan has been going through severe economic turmoil over the last 10 to 15 years.
- Note that the average return for treasury short rate denotes the average growth of the rate over the measurement period (theoretically should be averaged out and close to zero). Volatility of treasury short rates is surprisingly high. This is possibly due to drastic changes of the economic cycle over the measurement period (past 20 years).

As a next step, the return data are assumed to follow some pre-selected common statistical distributions, in order to yield the parameters for simulating future return scenarios.

Fund	Fitted distribution
Treasury short rate	One-factor Vasicek lognormal distribution with mean reversion
Stock funds	Regime-switching lognormal distribution with two regimes
Bond funds	Lognormal distribution

They are the most popular ones in modern finance theory for fitting historical market returns. In particular, a distribution with two regimes was chosen for stock funds, and it is generally believed that these two regimes would represent the optimistic and pessimistic parts of the economic cycle respectively. Each of these regimes is itself a lognormal distribution with its own average return and volatility, and an extra matrix of probabilities is introduced to govern the flow of the economic cycle (i.e. to control which regime that the outputs are subject to in each period). Since economic cycle should be a global parameter to be looked at, we have assumed that all stock funds are subject to the same matrix of probabilities and therefore the same regime for any particular future time period. Historical market data of the Hong Kong stock fund (i.e. Hang Seng Index) have been used for determining this matrix, as we assumed it being the most dominant index for mainly investors in Hong Kong.

It is important to note that only the risk premium part (i.e. return on top of treasury short rate) was modeled into the regime-switching distribution. Therefore, if one would like to find the future projected total return of a stock fund, it should be treasury rate from the Vasicek distribution plus the risk premium from the regime-switching distribution. On the contrary, total returns were modeled for bond funds, into the respective lognormal distributions.

For treasury short rate, a starting rate was chosen to be the actual short rate as at end of January 2007. Rates for subsequent future periods were then rolled-forward largely based on the volatility factor. Each rate after its roll-forward was adjusted by an extra function which tried to pull the rate back to the mean reversion target, to the extent of which was determined by the mean reversion strength. The determination of both the target and the strength were not empirical and were solely based on judgment.

(3) Parameters of fitted distributions

Matrix of transitional probabilities for stock funds

		To	
		State 1	State 2
From	State 1	95.444%	4.556%
	State 2	9.880%	90.120%
Terminal probability		68.442%	31.558%

Mean return and volatility parameters for stock funds & bond funds

Equity parameters						Bond parameters		
		Hong Kong Stock Fund	US Stock Fund	UK Stock Fund	Japan Stock Fund	EM Bond Fund	Global Trading Bond Fund	Global Fixed Income Bond
Regime 1	Expected Risk Premium	1.69%	1.29%	0.92%	0.54%	1.09%	0.43%	0.42%
	Volatility	4.97%	2.84%	3.07%	5.18%	3.95%	1.88%	0.44%
Regime 2	Expected Risk Premium	-1.29%	-1.25%	-0.86%	-1.73%	N/A	N/A	N/A
	Volatility	12.44%	6.04%	6.60%	7.69%	N/A	N/A	N/A

Parameters for cash or cash equivalent fund

Interest rate parameters	
	Treasury Short Rate
Starting rate	4.99%
Volatility	6.12%
Mean reversion target	4.00%
Mean reversion strength	0.80%

A few explanations on the output parameters:

- Method of maximum likelihood estimation (MLE) was used for determining the parameters of stock funds (2 global parameters – transitional probabilities from state 1 to state 2 and from state 2 to state 1; 4 local parameters for each of the 4 funds – ERP & volatility for each of the two regimes). In particular, the transitional probability matrix and the local parameters for Hong Kong stock fund were determined together from the historical Hang Seng Index data, during the first round of estimation. Local parameters for each of the other stock funds were then, in turn, determined during subsequent rounds of estimation, with the probability matrix fixed. In short, this determination using MLE can be thought of as testing different combinations of parameters in order to achieve a maximum goodness-of-fit.

- Details of the MLE method would not be provided here – they are definitely not the focus of this project. However, a number of desirable features of this method are worthwhile to note, especially to keep us rest assured that the fitted parameters are accurate.
  1. MLE has good statistical qualities and is generally thought to generate the "best" parameter estimates. Parameters determined using MLE are asymptotically efficient and unbiased.
  2. MLE adjusts for different cell sizes.
  3. MLE handles censored data. This data is censored at one month.

- The parameters for the stock funds are consistent with historical averages – the historical mean and volatility all lay between that of regime 1 and regime 2. For the bond funds, there is no curve fitting technique involved, and future returns are just assumed to be lognormal with mean and volatility equal to historical averages. For the cash fund, the starting rate of 4.99% is the actual rate as at January 2007, while the volatility parameter of 6.12% comes directly from historical data. Both of mean reversion target and strength are based on reasonable judgment.

- The transitional probabilities form a Markov matrix – the determination of regime in the current time period only depends on the regime in the previous time period. For example, if it is the first regime in month t, then it will have 95.4% probability of remaining in regime 1 and 4.6% probability of switching to regime 2 in month t+1. This matrix also gives rise to the concept of terminal probability – the proportion of time which the economy will be in regime 1 and regime 2 in the long run – 68.4% for regime 1 and 31.6% for regime 2. Having mentioned that these two regimes represent good and bad states of the economic cycle respectively, it makes sense that the economy would stay a significantly higher proportion of time in good economy (regime 1).

<u>(4) Generation of stochastic return scenarios using Monte-Carlo simulation</u>

Before simulation can be performed, a correlation coefficient matrix among the historical returns of the various cash, stock, and bond funds was first determined. It will be applied to the random variables during the simulation process, in order to factor in the same correlation relationships among the various funds into the simulated outputs as well.

	Treasury Short Rate	Hong Kong Stock Fund	US Stock Fund	UK Stock Fund	Japan Stock Fund	EM Bond Fund	Global Trading Bond Fund	Global Fixed Income Bond
Treasury Short Rate	1.0000	0.1098	0.1561	0.1821	0.1047	0.0791	(0.1091)	(0.1451)
Hong Kong Stock Fund	0.1098	1.0000	(0.0465)	0.5944	0.4352	(0.2203)	(0.0042)	(0.0240)
US Stock Fund	0.1561	(0.0465)	1.0000	0.0409	0.0655	(0.0577)	(0.0028)	0.0067
UK Stock Fund	0.1821	0.5944	0.0409	1.0000	0.4545	(0.0174)	0.0607	(0.0817)
Japan Stock Fund	0.1047	0.4352	0.0655	0.4545	1.0000	0.0405	(0.1443)	(0.0570)
EM Bond Fund	0.0791	(0.2203)	(0.0577)	(0.0174)	0.0405	1.0000	0.0577	(0.0013)
Global Trading Bond Fund	(0.1091)	(0.0042)	(0.0028)	0.0607	(0.1443)	0.0577	1.0000	0.0762
Global Fixed Income Bond	(0.1451)	(0.0240)	0.0067	(0.0817)	(0.0570)	(0.0013)	0.0762	1.0000

Monte-Carlo simulation was then performed via the following steps:

- For each scenario and for each projected future month, generate 9 random numbers of between 0 and 1. Quasi-random numbers are generated in this case instead of pseudo-numbers (ones resulting from Microsoft Excel RAND() function) since quasi-numbers are generally proven to be more robust and would result in minimum statistical bias.

- The first random number was used for determining the appropriate regime for the particular month, based on the regime of the previous month (also assuming the starting month was at regime 1), together with the transitional probability matrix. The other 8 random numbers, before they can be used to determine the projected monthly return for each of the 8 funds, were first transformed into standard normal random variables. After that, this vector of 8 standard normal numbers, was then multiplied by a square matrix (call it C for now) in order to yield 8 transformed normal numbers, such that they will then follow the correlation relationship consistent with the historical correlation coefficient matrix above.

- The determination of C was rather complex. In short, it involves solving a positive semi-definite matrix which is the "square root" of the historical correlation coefficient matrix above – that means (matrix C) times (transpose of C) equals historical matrix. The underlying theory (a mathematical technique known as Cholesky Decomposition) is not straightforward and would not be discussed in detail here.

- The 8 transformed normal numbers were then used to determine the projected monthly return for each of the 8 funds, for the particular projected future month and scenario. This process was then repeated 180,000 times (360 months times 500 scenarios) to yield the entire set of investment return scenarios.

(5) Discussion on limitations of the current approach

- Only 500 random scenarios were generated for this pricing exercise. While theoretically more scenarios would be better in terms of minimizing the scenario sampling error, generating more than 500 scenarios would not be feasible in this case due to runtime and system capability concerns, especially when our pricing was performed entirely in a Microsoft Excel environment. On the other hand, we believe that a set of 500 scenarios would already be enough to achieve reasonable accuracy, as we have tested the robustness of our scenario set using different sets of 500 scenarios and all of them have rendered close results as far as the cost of guarantee is concerned. In addition, the following table, which shows a comparison between the mean and volatility of historical and simulated returns for each of the funds, suggests that our simulated results are pretty close to the historical figures. The fact that the figures are not the same is because our methodology allows the separate modeling of the risk-free rate and risk premium which in turn leads to certain distortion when we compare the total return figures.

		Treasury Short Rate	Hang Seng Index	S&P 500 Index	FTSE 100 Index	Nikkei 225 Index	EM Bond	Global Bond	SSB Bond
Average of LN of monthly returns	From historical data	-0.04%	1.09%	0.88%	0.72%	0.19%	0.84%	0.43%	0.41%
	From simulated data	0.00%	0.84%	0.82%	0.69%	0.06%	1.00%	0.42%	0.42%
Average volatility of LN of monthly returns	From historical data	6.12%	8.30%	4.27%	4.59%	6.18%	4.25%	1.94%	0.45%
	From simulated data	6.55%	8.25%	4.29%	4.56%	6.19%	3.95%	1.88%	0.44%

- Historical return data of various stock market indices were used as a direct proxy for projecting future investment returns for the proposed funds included in our product. This implies that these stock funds can only be regarded as index tracking funds which mirror the returns of the respective market indices. Theoretically, if the funds offered in the product are not market index funds, alternative historical data sources (e.g. data from some commercial open-end funds) should be used as the starting point instead. Another quick way to get around, is to remain sticking to the historical index return data, but adjust the output scenarios afterwards using the CAPM approach (perform a separate regression to quantify the correlation between the market index and the particular fund being offered, in terms of beta estimate). However, neither of these has been performed in this project. We have accepted the notion that offering index tracking funds may already be good enough in this case, since it can be thought of as an investment restriction to limit the cost of guarantees (restricting investors from selecting funds more aggressive than the market portfolio).

- Same correlation relationships among various funds were assumed for both regimes (to equal historical correlations). This assumption is definitely not “right”, but may be the closest we can get given the time frame for this project (otherwise we may have to perform a statistical clustering of history into regime 1 and regime 2).

- The starting rate is crucial for the treasury short rate estimation but yet, it has been selected quite arbitrarily (actual as at January 2007). Theoretically the mean reversion target and strength parameters can pull the rate back to reasonable level in the long run, however, the determination of both were also not scientific (if not arbitrary).

Results, analysis & discussion

As mentioned earlier, our entire profit testing exercise was based on a single modal point which renders the following policy characteristics. While we have assumed that this modal point would represent the average profile of our entire portfolio should this product is going to be launched, details on justification of individual characteristics are already discussed in the previous section.

Issue Age	30
Sex	Male
Benefit Period	30 years
Premium Mode	Single
Premium Size	HKD 100,000

The primary focus of this profit testing is to assess the cost of various death and living benefit guarantees offered. In particular, we have profit tested the modal point with each type of guarantees chosen (GMDB & GMWB are first tested separately). Since the asset allocation into different funds would greatly affect the cost of guarantees, we have also tested the profit margin for investment into different funds as well as some mix of funds.

(1) Analysis with respect to pricing of GMDB

Gross Profit Margin for the product with various GMDB guarantees, assuming various asset allocations by investor

	No guarantee	Return of net consideration	2% Roll-up	4% Roll-up	Monthly Ratchet	2-year Ratchet	3-year Ratchet
(1) 100% Cash or cash equivalent	4.52%	4.52%	4.52%	4.51%	4.52%	4.52%	4.52%
(2) 100% Hong Kong stock fund	9.48%	9.41%	9.37%	9.31%	9.13%	9.26%	9.30%
(3) 100% US stock fund	9.08%	9.07%	9.06%	9.04%	9.00%	9.03%	9.04%
(4) 100% UK stock fund	8.89%	8.88%	8.87%	8.85%	8.80%	8.84%	8.85%
(5) 100% Japan stock fund	8.01%	7.81%	7.67%	7.44%	7.56%	7.69%	7.74%
(6) 100% Global trading bond fund	6.12%	6.12%	6.12%	6.10%	6.10%	6.11%	6.12%
(7) 100% Global fixed income fund	6.10%	6.10%	6.10%	6.10%	6.10%	6.10%	6.10%
(8) 100% Emerging market bond fund	6.73%	6.73%	6.73%	6.73%	6.71%	6.73%	6.73%
(9) Equal % allocation for each of 8 funds	7.54%	7.54%	7.54%	7.54%	7.51%	7.53%	7.54%
(10) Best estimate asset allocation	8.19%	8.19%	8.19%	8.18%	8.12%	8.16%	8.17%

The above table shows the gross profit margins (average across all stochastic scenarios) on the modal point, for different types of GMDB to be invoked, under different choices of asset allocation. There are a number of findings to be noted:

- The profit margins in the "no guarantee" column represent the profitability of the basic product (without any guarantee). Since we have not defined any investment restriction in this product design, the choice of how the assets are to be allocated into various funds is entirely up to policyholders. Therefore, we profit tested the policy under various possible asset allocations – 100% allocation in each of the 8 funds, an equal % allocation into each fund, as well as our best estimate of how policyholders are generally expected to allocate their assets. While the results based on best estimate allocation would represent our best estimate profitability of our portfolio, we believe that testing the profitability under these 100% single fund allocations can give us a better idea as to which funds are implying more guarantee costs.

- Our best estimate asset allocation is as follows:

(1) Cash or cash equivalent	5%
(2) Hong Kong stock fund	30%
(3) US stock fund	15%
(4) UK stock fund	15%
(5) Japan stock fund	5%
(6) Global trading bond fund	20%
(7) Global fixed income fund	5%
(8) Emerging market bond fund	5%

- This asset allocation implies a 65% investment into equity funds and a 35% investment into bond/cash funds. Intuitively this equity allocation may be a bit low; since we would normally expect rational policyholders investing in VA products would place capital appreciation as their primary objective and therefore maximize their holdings in higher growth funds. However, given the long investment horizon (30 years), we believe that many policyholders would return to a higher proportion of bond/cash funds towards the later part approaching retirement. Therefore, these expectations would average out to be our best estimate allocation above.

- Profit margins for the basic product ("no guarantee" column) fall into the range of 4% to 9% under various asset allocations, which are reasonable. Without the presence of any guarantee, it appears that profitability would only tie to the average return of assets (i.e. higher profitability for funds with higher mean returns), but not volatility. More charges will be collected in the case of higher returns, and volatility would not have any impact in this case since benefits are not asymmetric.

- In this product design, there is no additional charge for any type of GMDB to be elected. Therefore, profit margins all reduce as compared to the "no guarantee" case, though only to a negligible extent. Out of the various GMDB types, a monthly ratchet appears to be the only one that renders a noticeable impact – in the "worst" case it causes a 0.45% reduction for Japan stock fund.

- For easy reference, the following table shows the cost of each guarantee elected under each asset allocation, calculated as the difference between the “no guarantee” base case profit margin and the profit margin of the respective columns.

Cost of various guarantees elected
(calculated as difference between "no guarantee" profit margin & profit margin for electing the respective guarantee)

		Return of net consideration	2% Roll-up	4% Roll-up	Monthly Ratchet	2-year Ratchet	3-year Ratchet
(1) 100% Cash or cash equivalent		0.00%	0.00%	0.01%	0.00%	0.00%	0.00%
(2) 100% Hong Kong stock fund		0.06%	0.10%	0.16%	0.35%	0.22%	0.17%
(3) 100% US stock fund		0.01%	0.02%	0.04%	0.09%	0.05%	0.04%
(4) 100% UK stock fund		0.01%	0.02%	0.05%	0.09%	0.06%	0.05%
(5) 100% Japan stock fund		0.20%	0.35%	0.57%	0.45%	0.32%	0.27%
(6) 100% Global trading bond fund		0.00%	0.00%	0.02%	0.02%	0.01%	0.01%
(7) 100% Global fixed income fund		0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
(8) 100% Emerging market bond fund		0.00%	0.00%	0.00%	0.03%	0.01%	0.00%
(9) Equal % allocation for each of 8 funds		0.00%	0.00%	0.00%	0.03%	0.01%	0.01%
(10) Best estimate asset allocation		0.00%	0.00%	0.01%	0.07%	0.03%	0.02%

- This analysis shows that the presence of any GMDB (at least for the types within our analysis) would not incur significant cost to the company. Of course, this resulting negligible impact is partly due to the fact that relatively younger ages are being considered for our selected modal point – from age 30 to age 60. As mortality cost increases with age, we would expect the impact will be more noticeable for older ages but this is already beyond our project scope. Therefore, in this case, we can conclude that offering a GMDB with no additional charge would not have any significant financial impact to the company.

- A table showing the various percentiles of profit margins among the stochastic scenarios (in contrast to the mean in the above table) is shown below for reference.

Various Percentiles of Gross Profit Margin for the product with various GMDB guarantees, assuming various asset allocations by investor

		No guarantee	Return of net consideration	2% Roll-up	4% Roll-up	Monthly Ratchet	2-year Ratchet	3-year Ratchet
	10th percentile	4.41%	4.41%	4.41%	4.36%	4.41%	4.41%	4.41%
	25th percentile	4.44%	4.44%	4.44%	4.42%	4.44%	4.44%	4.44%
(1) 100% Cash or cash equivalent	50th percentile	4.50%	4.50%	4.50%	4.50%	4.50%	4.50%	4.50%
	75th percentile	4.58%	4.58%	4.58%	4.58%	4.58%	4.58%	4.58%
	90th percentile	4.65%	4.65%	4.65%	4.65%	4.65%	4.65%	4.65%
	10th percentile	8.04%	7.91%	7.81%	7.63%	7.40%	7.57%	7.59%
	25th percentile	8.71%	8.69%	8.67%	8.64%	8.44%	8.54%	8.59%
(2) 100% Hong Kong stock fund	50th percentile	9.51%	9.51%	9.51%	9.51%	9.28%	9.39%	9.41%
	75th percentile	10.21%	10.21%	10.21%	10.20%	10.06%	10.13%	10.14%
	90th percentile	10.97%	10.97%	10.97%	10.97%	10.74%	10.86%	10.87%
	10th percentile	8.29%	8.28%	8.26%	8.19%	8.18%	8.21%	8.23%
	25th percentile	8.68%	8.68%	8.67%	8.66%	8.56%	8.60%	8.62%
(3) 100% US stock fund	50th percentile	9.09%	9.09%	9.09%	9.09%	9.03%	9.06%	9.06%
	75th percentile	9.52%	9.52%	9.52%	9.52%	9.47%	9.50%	9.51%
	90th percentile	9.89%	9.89%	9.89%	9.89%	9.84%	9.89%	9.89%
	10th percentile	8.14%	8.12%	8.09%	8.00%	7.98%	8.02%	8.05%
	25th percentile	8.47%	8.47%	8.47%	8.45%	8.39%	8.41%	8.44%
(4) 100% UK stock fund	50th percentile	8.91%	8.91%	8.90%	8.90%	8.83%	8.86%	8.87%
	75th percentile	9.29%	9.29%	9.29%	9.29%	9.24%	9.27%	9.27%
	90th percentile	9.69%	9.69%	9.69%	9.69%	9.64%	9.67%	9.67%
	10th percentile	6.93%	6.36%	6.07%	5.52%	6.08%	6.37%	6.43%
	25th percentile	7.54%	7.28%	7.05%	6.77%	7.01%	7.18%	7.22%
(5) 100% Japan stock fund	50th percentile	8.06%	7.99%	7.90%	7.72%	7.77%	7.84%	7.88%
	75th percentile	8.61%	8.60%	8.58%	8.55%	8.44%	8.50%	8.51%
	90th percentile	9.01%	9.00%	9.00%	8.99%	8.83%	8.90%	8.93%
	10th percentile	5.90%	5.90%	5.89%	5.82%	5.88%	5.89%	5.90%
	25th percentile	6.01%	6.01%	6.01%	5.98%	5.99%	6.00%	6.01%
(6) 100% Global trading bond fund	50th percentile	6.13%	6.13%	6.13%	6.13%	6.11%	6.12%	6.13%
	75th percentile	6.22%	6.22%	6.22%	6.22%	6.21%	6.22%	6.22%
	90th percentile	6.32%	6.32%	6.32%	6.32%	6.31%	6.32%	6.32%
	10th percentile	6.05%	6.05%	6.05%	6.05%	6.05%	6.05%	6.05%
	25th percentile	6.07%	6.07%	6.07%	6.07%	6.07%	6.07%	6.07%
(7) 100% Global fixed income fund	50th percentile	6.10%	6.10%	6.10%	6.10%	6.10%	6.10%	6.10%
	75th percentile	6.13%	6.13%	6.13%	6.12%	6.12%	6.13%	6.13%
	90th percentile	6.15%	6.15%	6.15%	6.15%	6.14%	6.15%	6.15%
	10th percentile	6.31%	6.31%	6.31%	6.31%	6.28%	6.31%	6.31%
	25th percentile	6.53%	6.53%	6.53%	6.53%	6.50%	6.52%	6.52%
(8) 100% Emerging market bond fund	50th percentile	6.73%	6.73%	6.73%	6.73%	6.71%	6.73%	6.73%
	75th percentile	6.94%	6.94%	6.94%	6.94%	6.91%	6.93%	6.94%
	90th percentile	7.15%	7.15%	7.15%	7.15%	7.12%	7.14%	7.15%
	10th percentile	6.93%	6.93%	6.93%	6.93%	6.92%	6.93%	6.93%
	25th percentile	7.19%	7.19%	7.19%	7.19%	7.17%	7.18%	7.18%
(9) Equal % allocation for each of 8 funds	50th percentile	7.54%	7.54%	7.54%	7.54%	7.52%	7.53%	7.54%
	75th percentile	7.89%	7.89%	7.89%	7.89%	7.86%	7.88%	7.88%
	90th percentile	8.19%	8.19%	8.19%	8.19%	8.15%	8.17%	8.18%
	10th percentile	7.40%	7.40%	7.39%	7.38%	7.36%	7.38%	7.39%
	25th percentile	7.76%	7.76%	7.76%	7.75%	7.71%	7.73%	7.74%
(10) Best estimate asset allocation	50th percentile	8.22%	8.22%	8.22%	8.22%	8.16%	8.20%	8.22%
	75th percentile	8.64%	8.64%	8.64%	8.64%	8.54%	8.59%	8.60%
	90th percentile	8.98%	8.98%	8.98%	8.98%	8.90%	8.94%	8.95%

(2) Analysis with respect to pricing of GMWB

Gross Profit Margin for the product with various GMWB guarantees, assuming various asset allocations by investor

	No guarantee	Return of net consideration	2% Roll-up	4% Roll-up	Monthly Ratchet	2-year Ratchet	3-year Ratchet
(1) 100% Cash or cash equivalent	4.52%	9.64%	9.64%	8.80%	9.56%	9.57%	9.57%
(2) 100% Hong Kong stock fund	9.48%	8.26%	4.90%	-0.54%	-17.15%	-6.91%	-3.70%
(3) 100% US stock fund	9.08%	12.75%	11.92%	10.41%	6.69%	9.10%	9.95%
(4) 100% UK stock fund	8.89%	12.62%	11.64%	9.71%	5.64%	8.17%	8.97%
(5) 100% Japan stock fund	8.01%	-7.46%	-21.17%	-44.34%	-35.22%	-21.73%	-16.75%
(6) 100% Global trading bond fund	6.12%	10.98%	10.81%	9.50%	9.31%	10.09%	10.29%
(7) 100% Global fixed income fund	6.10%	10.97%	10.97%	10.93%	10.91%	10.97%	10.97%
(8) 100% Emerging market bond fund	6.73%	11.77%	11.75%	11.68%	10.09%	11.21%	11.40%
(9) Equal % allocation for each of 8 funds	7.54%	12.30%	12.29%	12.15%	10.34%	11.41%	11.66%
(10) Best estimate asset allocation	8.19%	12.88%	12.75%	12.34%	10.50%	11.50%	11.90%

Figures in the "no guarantee" column, again, represent profitability of the basic product and are the same as that of the GMDB table. For the election of different GMWB guarantees, the following can be observed:

- In this product design, an additional charge of 8 basis points per month (i.e. an annual charge of 96 basis points of AV) is collected upon the election of any GMWB guarantee. We have set the additional charge at this level since, in our opinion, this is already close to the maximum level which policyholders would be willing to pay for this kind of guarantee in the Hong Kong market. It is observed that a few of these GMWB guarantees are affordable by this level of guarantee charge, while others are not.

- It can be noted that a profit margin that is higher than the corresponding "no guarantee" profit margin would indicate that the additional guarantee charge is more than enough to fund the cost of guarantee. For example, in the case of 100% cash allocation, the guarantee charge should be more than enough to fund any types of GMWB, since all the profit margins are higher than the base case profit margin of 4.52%.

- For all the 100% cash/bond allocations ((1), (6), (7) & (8) above), the guarantee charge is more than enough to fund the respective guarantee in all cases. The reason is probably due to the relatively low volatility of these funds. In particular, the global trading bond fund produces the lowest profit margins among these 4 funds, rather than the intuitive guess of the emerging market bond fund which has the highest volatility. The reason is due to the effect of the dynamic lapse rate formula which has caused relatively higher proportion of people in the emerging market bond fund to lapse in earlier durations; before the cost of guarantee becomes excessively high (EM bond fund has a higher benchmark return than global bond fund in the dynamic lapse formula).

- For the 100% stock fund allocations ((2), (3), (4) & (5) above), it can be observed that not all types of GMWB can be afforded by the current level of guarantee charge. The results are better for the US stock fund and the UK stock fund, where only the monthly ratchet guarantee would cause a negative financial impact on the basic product (6.69% & 5.64% as compared to base profit margins of 9.08% & 8.89%, for the US & UK funds respectively). But yet, this degree of profitability reduction should still be acceptable, provided that we can still obtain a profit margin of above 5%.

- For the Hong Kong stock fund, both the presence of monthly ratchet, 2-year ratchet, 3-year ratchet and 4% roll-up GMWB would cause detrimental impact to the basic product, in which each of them even eats up the profitability of the basic product to result in a negative profit margin for the aggregate product. On the other hand, the 2% roll-up and return of net considerations GMWB can more or less retain the profitability of the basic product.

- Results for the Japan stock fund are definitely the worst, since all the GMWB guarantees would result in negative profit margins for the aggregate product. In overall, the results of these stock funds are close to our expectation that high cost of guarantee will be exhibited by funds with higher volatility.

- In deciding which types of GMWB should be made available, considerations should not be solely based on the implied profitability of the best estimate asset allocation, which seems to suggest that all types should be affordable. We definitely would not desire policies with 100% cash allocation to subsidize policies with 100% stock allocation, therefore it is better to make sure the guarantee charge is adequate under any asset allocation (where possible). One way to revamp the product design is to collect different levels of guarantee charge for investments into different funds, in order to reflect the difference in the cost of guarantees implied. However, this may not be always possible due to marketing concern and also system limitations to handle the complications caused by different guarantee charge levels. Another way is to limit the types of guarantee offered, such that guarantees with excessive cost implications will then be eliminated. A third way would be to restrict the range of funds to be offered, such that funds with excessive risks are not offered to policyholders (or allocation to these high risk funds is limited to only certainly percentage).

- For easy reference, the following table shows the cost of each guarantee elected under each asset allocation, calculated as the difference between the "no guarantee" base case profit margin and the profit margin of the respective columns. Note that for this comparison we have assumed the 0.8% of monthly guarantee charge is also deducted in the "no guarantee" case so as to render an apple-to-apple comparison.

Cost of various guarantees elected
(calculated as difference between "no guarantee" profit margin & profit margin for electing the respective guarantee)

		Return of net consideration	2% Roll-up	4% Roll-up	Monthly Ratchet	2-year Ratchet	3-year Ratchet
(1) 100% Cash or cash equivalent		0.00%	0.00%	0.84%	0.08%	0.07%	0.06%
(2) 100% Hong Kong stock fund		5.92%	9.28%	14.72%	31.33%	21.09%	17.88%
(3) 100% US stock fund		0.95%	1.78%	3.29%	7.01%	4.60%	3.75%
(4) 100% UK stock fund		0.86%	1.83%	3.76%	7.83%	5.30%	4.51%
(5) 100% Japan stock fund		19.90%	33.61%	56.78%	47.66%	34.17%	29.19%
(6) 100% Global trading bond fund		0.02%	0.19%	1.50%	1.69%	0.91%	0.71%
(7) 100% Global fixed income fund		0.00%	0.00%	0.04%	0.06%	0.00%	0.00%
(8) 100% Emerging market bond fund		0.01%	0.03%	0.10%	1.69%	0.57%	0.38%
(9) Equal % allocation for each of 8 funds		0.00%	0.02%	0.16%	1.96%	0.89%	0.64%
(10) Best estimate asset allocation		0.03%	0.15%	0.56%	2.40%	1.41%	1.01%

- From this analysis, we believe that offering a monthly ratchet may be a bit dangerous, since it may result in excessive negative profitability for some particular stock fund allocations. All the others seem acceptable. Therefore, we recommend offering a 4% roll-up as well as a 2-year ratchet GMWB guarantee, since they are the most competitive out of the rest.

- A table showing the various percentiles of profit margins among the stochastic scenarios (in contrast to the mean in the above table) is shown below for reference. They obviously show a wider range of profit margins compared to the corresponding table for GMDB. It is obvious that when looking at the extreme percentiles (e.g. 95%) instead of the averages, the costs of guarantees would then be a lot higher as compared to the base profit margin.

Various Percentiles of Gross Profit Margin for the product with various GMWB guarantees, assuming various asset allocations by investor

		No guarantee	Return of net consideration	2% Roll-up	4% Roll-up	Monthly Ratchet	2-year Ratchet	3-year Ratchet
(1) 100% Cash or cash equivalent	10th percentile	4.41%	9.49%	9.49%	6.56%	9.31%	9.32%	9.34%
	25th percentile	4.44%	9.53%	9.53%	8.35%	9.49%	9.50%	9.51%
	50th percentile	4.50%	9.61%	9.61%	9.61%	9.60%	9.60%	9.60%
	75th percentile	4.58%	9.72%	9.72%	9.72%	9.71%	9.71%	9.71%
	90th percentile	4.65%	9.82%	9.82%	9.82%	9.82%	9.82%	9.82%
(2) 100% Hong Kong stock fund	10th percentile	8.04%	1.17%	-6.01%	-22.53%	-68.15%	-43.48%	-32.04%
	25th percentile	8.71%	12.77%	11.64%	7.10%	-18.04%	-7.03%	-2.43%
	50th percentile	9.51%	14.20%	14.17%	14.07%	1.30%	6.56%	8.93%
	75th percentile	10.21%	15.06%	15.06%	15.06%	10.36%	12.45%	13.49%
	90th percentile	10.97%	15.94%	15.94%	15.94%	14.96%	15.15%	15.43%
(3) 100% US stock fund	10th percentile	8.29%	11.68%	9.19%	4.76%	-2.00%	1.57%	3.65%
	25th percentile	8.68%	13.20%	13.14%	12.68%	5.13%	7.76%	8.65%
	50th percentile	9.09%	13.72%	13.71%	13.69%	9.80%	11.49%	12.36%
	75th percentile	9.52%	14.21%	14.21%	14.21%	11.92%	13.25%	13.57%
	90th percentile	9.89%	14.68%	14.68%	14.68%	13.35%	14.21%	14.40%
(4) 100% UK stock fund	10th percentile	8.14%	10.74%	7.39%	1.49%	-3.30%	0.57%	1.91%
	25th percentile	8.47%	12.93%	12.79%	10.92%	3.38%	6.82%	7.70%
	50th percentile	8.91%	13.49%	13.47%	13.40%	8.78%	11.08%	11.75%
	75th percentile	9.29%	13.94%	13.94%	13.94%	11.65%	12.93%	13.34%
	90th percentile	9.69%	14.42%	14.42%	14.42%	12.93%	13.92%	14.09%
(5) 100% Japan stock fund	10th percentile	6.93%	-41.88%	-76.51%	-136.97%	-102.76%	-72.34%	-55.95%
	25th percentile	7.54%	-11.34%	-28.37%	-56.43%	-46.22%	-28.66%	-20.99%
	50th percentile	8.06%	8.31%	0.02%	-13.48%	-16.12%	-7.19%	-4.36%
	75th percentile	8.61%	13.11%	12.88%	10.32%	0.35%	4.55%	6.16%
	90th percentile	9.01%	13.61%	13.60%	13.50%	6.64%	9.89%	10.71%
(6) 100% Global trading bond fund	10th percentile	5.90%	10.73%	10.41%	5.64%	7.41%	8.58%	8.96%
	25th percentile	6.01%	10.87%	10.84%	9.15%	8.97%	9.90%	10.08%
	50th percentile	6.13%	11.02%	11.01%	10.94%	9.75%	10.49%	10.68%
	75th percentile	6.22%	11.13%	11.13%	11.13%	10.23%	10.83%	10.96%
	90th percentile	6.32%	11.24%	11.24%	11.24%	10.52%	11.07%	11.14%
(7) 100% Global fixed income fund	10th percentile	6.05%	10.91%	10.91%	10.81%	10.83%	10.91%	10.91%
	25th percentile	6.07%	10.94%	10.94%	10.94%	10.87%	10.94%	10.94%
	50th percentile	6.10%	10.97%	10.97%	10.97%	10.92%	10.97%	10.97%
	75th percentile	6.13%	11.00%	11.00%	11.00%	10.95%	11.00%	11.00%
	90th percentile	6.15%	11.03%	11.03%	11.03%	10.98%	11.03%	11.03%
(8) 100% Emerging market bond fund	10th percentile	6.31%	11.27%	11.27%	11.27%	8.61%	9.98%	10.46%
	25th percentile	6.53%	11.52%	11.52%	11.52%	9.56%	10.90%	11.14%
	50th percentile	6.73%	11.79%	11.79%	11.79%	10.41%	11.50%	11.61%
	75th percentile	6.94%	12.02%	12.02%	12.02%	10.92%	11.86%	11.93%
	90th percentile	7.15%	12.29%	12.29%	12.29%	11.42%	12.17%	12.23%
(9) Equal % allocation for each of 8 funds	10th percentile	6.93%	11.66%	11.65%	11.41%	8.34%	9.98%	10.45%
	25th percentile	7.19%	11.92%	11.92%	11.91%	9.76%	11.12%	11.42%
	50th percentile	7.54%	12.30%	12.30%	12.30%	10.80%	11.71%	11.91%
	75th percentile	7.89%	12.68%	12.68%	12.68%	11.51%	12.24%	12.38%
	90th percentile	8.19%	13.00%	13.00%	13.00%	11.96%	12.66%	12.77%
(10) Best estimate asset allocation	10th percentile	7.40%	12.03%	12.00%	10.74%	3.22%	6.60%	7.58%
	25th percentile	7.76%	12.44%	12.44%	12.32%	7.64%	9.93%	10.73%
	50th percentile	8.22%	12.95%	12.95%	12.95%	9.88%	11.40%	11.90%
	75th percentile	8.64%	13.38%	13.38%	13.38%	11.33%	12.51%	12.77%
	90th percentile	8.98%	13.78%	13.78%	13.78%	12.32%	13.23%	13.41%

(3) Analysis of the effect of waiting period

Currently the waiting period before the elected guarantee can be invoked is set to 5 years and 8 years for GMDB and GMWB respectively. Since it has already been shown that the presence of any GMDB would not incur any significant cost to the company, we believe that setting the waiting period for GMDB is also not a concern. Therefore, this section has focused on analyzing the effect of waiting period on GMWB, and we have assumed that no GMDB has been elected throughout this analysis.

Profit Margins for various waiting periods (assuming no GMDB elected)

	Best estimate asset allocation		100% Hong Kong stock fund	
GMWB elected	4% Roll-up	2-year Ratchet	4% Roll-up	2-year Ratchet
Base Case (8 years waiting period)	12.34%	11.50%	-0.54%	-6.91%
No waiting period	10.46%	10.44%	-13.68%	-15.85%
1 year waiting period	10.60%	10.44%	-13.18%	-15.85%
5 years waiting period	11.77%	10.72%	-5.90%	-13.35%
10 years waiting period	12.51%	11.86%	1.84%	-3.45%

As expected, profit margin increases when there is a longer waiting period, since the presence of a waiting period can limit the cost of a GMWB guarantee. For the best estimate asset allocation, it can be observed that the effect is actually not that big – only about 2% difference in profit margin for the presence of a 10-year waiting period, as compared to no waiting period. For the 100% Hong Kong stock fund allocation, the effect of a waiting period is relatively significant – causing a 14% increase in profit margin from the case of no waiting period to 10-year waiting period.

We believe that the positive effect brought by a longer waiting period is actually offset by an opposing effect – while few people would choose to withdrawal their policies during the waiting period (especially in poor investment environment), the presence of a longer waiting period will actually force people invoke the guarantee in later policy years, which may result in even higher guarantee cost compared to early withdrawal. From a modeling perspective, our dynamic lapse rate function has accounted for this effect, since the dependency strength is set to be lower before the waiting period (therefore resulting in lower lapse during poor investment environment).

Hence, for the best estimate allocation, we can conclude that while there is some positive effect brought by a waiting period, the impact would not be that large.

(4) Robustness of the dynamic lapse rate function

From previous analysis, it has been shown that the presence of a GMWB option can incur significant guarantee cost to the company. In addition, the cost that is incurred by a GMWB option would, in turn, be linked to the level of withdrawal. Therefore, it is then important to make sure that the lapse assumption during our profit test is accurate (or at least reasonable and adequate); otherwise we may end up offering guarantees which are not affordable by the current level of charges.

While it may not be possible to test the "accuracy" of our lapse assumption, in this analysis we approach the problem from a different angle and test its "robustness" instead. In particular, we sensitivity test the profit margins based on some altered lapse rate function and then compare them with the base case profit margin. This is to test how far the resulting profitability can change should our lapse rate function does not work so well in terms of predicting actual withdrawal.

In the earlier section, we defined the following dynamic lapse rate function:

Base rate + [benchmark return – (annualized return for last 3 years, adjusted for guarantee)] * dependency strength

The base rate, representing the static component, is a schedule which varies by policy year. The remaining part of the formula represents the dynamic component, which causes withdrawal to increase when actual investment performance (after taking into account the guaranteed return) is poor while causes withdrawal to decrease when performance is good. In particular, its effect on lapse rate is measured against a benchmark return, which represents the desired return that a rational investor would normally expect (varies by fund). In addition, the dependency strength controls the degree to which lapse rate is affected by investment performance. Obviously, this strength would be higher for stock funds and lower for other funds.

Profit Margins for various alternative lapse assumptions
(assuming 4% roll-up GMDB elected)

	Best estimate asset allocation		100% Hong Kong stock fund	
GMWB elected	4% Roll-up	2-year Ratchet	4% Roll-up	2-year Ratchet
Base Case	12.33%	11.49%	-0.72%	-7.09%
Base Rate increased by 20%	9.86%	9.07%	0.07%	-4.62%
Base Rate decreased by 20%	15.69%	14.82%	-1.86%	-10.46%
Dependency Strength doubled	12.51%	11.75%	0.04%	-8.26%
Without Dynamic Component	12.14%	10.74%	-3.30%	-6.25%

The above table shows the resulting profit margins for different lapse scenarios. For the scenarios in which the base rate is increased or decreased, results are as expected that higher lapses would cause higher cost of guarantees and hence lead to lower profit margins. However, the results are a bit counter-intuitive for the other two scenarios – profit margins are higher when the effect of the dynamic lapse component is doubled, while margins are lower when the dynamic component is removed.

We believe that the results are, again, affected by two opposing forces – when there is a dynamic lapse formula, people will tend not to lapse when the guarantee is in-the-money, thus resulting in higher cost of guarantee. However, when people choose to stay longer in the product, the profitability of the basic product will, on the other hand, increase; since charges are being collected for more number of years. As a result, the results shown above can be somewhat explained by the fact that the second effect is larger than the first effect, thereby increasing the profitability in the case of dynamic lapse.

Discussion on possible enhancements, and a look into other dimensions

<u>Reserving</u>: in generally it would be too difficult to take care of the reserving methodology as well during the pricing stage, when GMDB and GMAB are concerned; otherwise it would require a nested stochastic (also called "stochastic-in-stochastic") projection in order to stochastically determine the adequate reserve amount for each generated real-world scenario. In the US, what companies generally seek as an alternative is the Actuarial Guideline 34 approach – assuming the account value to suffer an immediate hit followed by subsequent recovery in future years when determining reserve with respect to a specific time point – this hypothetical stream of investment returns would then avoid stochastic projections at the second level.

Another alternative is to take the reserve amount as the aggregate value of the series of derivatives that are used to back the guarantee offered. VA writers generally adopt the first approach when reserving for GMDB and the second approach when reserving for GMAB.

<u>Market-consistent pricing</u>: in this project what we have performed is referred to as real-world pricing, which discounts the various cost of guarantees at each future time points using a company specific "hurdle rate". This is the type of pricing which we have adopted for so many years in the traditional actuarial literature. It works well for normal insurance products which do not render significant options to policyholders. In the case of pricing a guarantee, some people have started to argue that this real-world approach may not be so appropriate, because it ignores its relationships with other options that are publicly available in the open market.

Market-consistent technique bridges the gap between the value of guarantees offered in our insurance product and the value of related market options. In particular, it allows us to calculate the price of a guarantee as if this guarantee is being sold in the open market – it determines its reasonable price such that no arbitrage opportunity will arise.

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Please note that the following list contains the references that the project has been based upon which may not be exhaustive.

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