Pricing Variable Annuity
With Embedded Guarantees
- a case study

David Wang, FSA, MAAA
May 21, 2008 at ASHK
Set The Stage

- Peter is the pricing actuary of company LifeGoesOn and LifeGoesOn wishes to launch a product offering single premium protected retirement benefit. (DA with guarantees)
- Currently LifeGoesOn has some existing base variable annuity or unit-linked type of products.
- After talking to the Marketing, Peter has decided to introduce a lifetime GMWB (GLWB)
- Challenge: how to price a GLWB?
- Solution 1: add 10bp to what LifeNeverEnds charges
  - Very tempting, but .....
What are friends for

- Peter’s best friend, David, happens to have worked on this before, and he calls up David for help, despite still owing David $100 losing the bet on Houston v. Utah play-off results.
- “Don’t worry, Peter. What are friends for” David offers help immediately.
- “These are things you need to consider.” says the wise David.
Option pricing

- “GLWB is essentially a Put Option written by the company to policyholders on the policyholder account.

- The Put Option bites the company when account value goes to 0 and the policyholder collects a lifetime annuity afterwards.

- To price a Put Option, you could
  - Use closed form solution, perhaps possible for GMAB but unlikely for GMWB
  - Use stochastic simulation, price being the average of present value of projected stochastic claims incurred
Deterministic v. Stochastic

- Typically the base VA is priced on deterministic basis, with some sensitivity tests
  - If no GMDB, fine. If with GMDB, cost of GMDB tends to be small.

- With a GLWB rider, a deterministic best estimate of future fund growth is likely to result in no or little GLWB claims. Sensitivity tests can only help understand what happens if ..., but not the true costs

- Therefore, stochastic simulation is the way to go.
Risk Neutral v. Real World

- Option pricing, whether closed form solution or simulation, is done on risk neutral basis.
- Real world basis recognizes the risk premium included in the future fund growth, and in other words the price calculated accordingly is one where the writer takes in all the market risk.
- People are generally risk-averse and want to be compensated for risk taken, but the risk premium to assume can be subjective and vary by degree of risk aversion.
Risk Neutral v. Real World

- Alternatively, we could change the probabilities of future asset movements so that it looks like people are not asking a compensation for risk. In other words, we can model as if people are risk-neutral and all assets earn risk free rates, where the risk premium is embedded in the revised probabilities.

- Risk Neutral pricing calculates a price that assumes all market risks are hedged, (hedge cost)

- Risk neutral pricing forms the basis for prices of traded derivatives (market consistent)
Risk Neutral v. Real World

- Real World Scenarios can still be relevant depending on reporting
- Statutory (GAAP) profits on the entire contract
- Capital and reserve assessment (C3Phase 2, VACARVM)

- Under fair value accounting or MC NBV, then all on risk neutral basis
Number of Stochastic Scenarios

- Balance between run time and accuracy
  - Std error reduce by sqrt (times increase in No. Scenarios)
- Some convergence tests are recommended on test cell
Key Drivers of GLWB cost

- **Demographic**
  - Sales mix (cost varies by cell)
  - Lapses (fewer lapses higher costs)
  - Mortality
  - Fund allocation (cost varies by fund)

- **Economic**
  - Model
  - Interest
  - Volatility
Software

- The pricing software needs to be able to handle stochastic projection
- When moving to profit testing entire contract, nested stochastic projection might also be necessary
- Speed
- Ease of producing and analyzing results/reports
Subtlety

- The GLWB charge is also a drag on fund growth, and therefore affects the cost to be calculated.
- Theoretically, solve for a charge that equals the cost.
- Practically, charge is typically higher than cost to cover some unexpected non-market risk or purely as profit, and often times charge is driven by competitive needs. So the analysis is whether the expected charge is enough to cover cost calculated.”
Action!

- Peter gets off the phone, promised David to pay $100 soon, and makes a list of action plan.
- Decide product spec and assumptions
- Generate risk neutral economic scenarios
- Price GLWB rider to assess the sufficiency of the expected charge
  - Sensitivity on number of scenarios
  - Cell level
  - Lapse
  - Mortality
  - Fund allocation
  - Scenarios
Action!

- Also has to profit test the entire contract to meet the ROA target
  - Project VACARVM reserve and C3Phase 2 Capital (nested stochastic simulation)
- CRO wants to know the impact of hedging, and thus will have to perform simulation of dynamic hedge and study the impact
- Lastly, all these perhaps too much to ask for Excel, with the help of David, Peter decides to use MG-ALFA®
Product Specification

- **Base contract:**
  - Back-end load with SC for the first 7 years,
  - M&E charge of 110 bp
  - Minimum DB of ROP (reduced by PW proportionately)

- **GLWB**
  - Before first withdrawal, benefit base is max(5% compound rollup, annual ratchet)
  - After withdrawal, benefit base is annual ratchet based
  - Guaranteed Benefit %
    - 5% “for life” at ages 60-69
    - 6% “for life” at ages 70-79
    - 7% “for life” at ages 80-85
    - Attained age at first withdrawal sets maximum benefit percent.
Assumptions

- **Sales Related**
  - Average Initial Premium: $50,000
  - Gender mix: 50/50
  - GLWB election: 100% (conservative)
  - Issue Age and Deferral Period (bonus period)

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<thead>
<tr>
<th>Issue Age</th>
<th>0-yr Deferral</th>
<th>5-yr Deferral</th>
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<tr>
<td>85</td>
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<td>0%</td>
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</tr>
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Assumptions

- **Actuarial**
  - *Mortality:* 80% of Annuity 2000 Basic
  - *Base Lapse:* 1.5%, 2.5%, 3.0%, 3.0%, 4.0%, 5.0%, 8.0%, 40.0%, 10.0%, ...
  - Dynamic Lapse Function: (Important and Customized!)
    - When the GLWB is more in-the-money, the Dynamic Lapse Multiple will be less than 100% and thus, when applied to the Lapse Rate, will reduce Lapses.
    - Dynamic Lapse Multiple = Max [10%, 1 - 75%*(GLWB ITM% - 1.1)] if GLWB ITM% > 1.1, or 1 otherwise
    - GLWB ITM% = [PV of GLWB / AV]
Assumptions

- **Others**
  - GLWB charge: 60 bp of benefit base

<table>
<thead>
<tr>
<th>Assumed Fund Allocation</th>
<th>S&amp;P 500</th>
<th>Russell 2000</th>
<th>NASDAQ</th>
<th>SB BIG (Bond)</th>
<th>EAFE (International Equity)</th>
<th>Money Market</th>
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**Correlation Matrix**

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<tr>
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<th>S&amp;P500</th>
<th>Russell 2000</th>
<th>NASDAQ</th>
<th>SBBIG</th>
<th>EAFE</th>
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<td>0.62</td>
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</table>

**Long Term Volatility**

|          | 0.171 | 0.208 | 0.281 | 0.038 | 0.157 | 0.01 |

**Continuously compounded risk free rate**

4.50%
Scenario generation

- Geometric Brownian for fund growth
- Unlike GMIB, which is in its nature a fixed income derivative, GLWB is an equity derivative with payoff triggered by equity fund level becoming 0. Hence, ok to use an equity only model
- Risk Free Rates and Volatility assumption
  - Market consistent requires use of current swap curve and implied volatilities from traded European options
  - But no traded long dated European options
  - What to assume the long term
    - Historical average
    - Grading to historical average
## Pricing GLWB – No of Scenarios

### Test of Number of Stochastic Scenarios

**Table 5.3.1 Comparison between different number of scenarios**

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<td><strong>Weighted Average</strong></td>
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</table>
Pricing GLWB – No of Scenarios

- Costs of GLWB either on individual cell level or weighted average level do not change significantly when number of scenarios reduce from 10,000 to 1,000 or even to 500.

- Interestingly, results from 500 scenarios match those from 10,000 scenarios better. A test of running a different set of 500 scenarios shows that the weighted average cost is 44.16.

- Therefore, to have relatively higher credibility in the results, Peter decides to use 1,000 scenarios in future runs.
Pricing GLWB – Cell level

- Cell level
  - The younger the higher cost (more likely to live to get the guarantee)
  - Cost also varies by bonus period, but the extra bonus does not always increase costs. At 10 year maximum possible bonus, cost is in fact lower (no withdrawal helps the growth the fund account)
Pricing GLWB - Lapses

- Lapses
  - Sensitivity of dynamic lapse multiplier
  - Assumed: Max [10%, 1 - 75% \times (GLWB ITM\% - 1.1)] when GLWB ITM\% is greater than 110%
  - Sensitivity: Max [0, 1 - 100\% \times (GLWB ITM\% - 1)] if GLWB ITM\% is greater than 100%
Pricing GLWB - Lapses

- **Lapses**
  - The sensitivity dynamic lapse assumption increases the costs of GLWB by up to 20%, but on average the expected charge of 60 basis points is still adequate.

| Table 5.3.2.2 Comparison between assumed dynamic lapses and sensitivity dynamic lapses |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| Assumed Dynamic Lapses            | Sensitivity Dynamic Lapses        | Assumed Dynamic Lapses            | Sensitivity Dynamic Lapses        |
| Female                            |                                 | Female                            |                                 |
| Age                               | Deferred Period                  | Age                               | Deferred Period                  |
| N/A                               | 0                                | N/A                               | 0                                |
| 55                                | 78.2                             | 55                                | 93.5                             |
| 65                                | 77.9                             | 65                                | 90.5                             |
| 75                                | 48.7                             | 75                                | 54.7                             |
| 85                                | N/A                              | 85                                | 31.2                             |
| Male                              |                                  | Male                              |                                  |
| Age                               | Deferred Period                  | Age                               | Deferred Period                  |
| N/A                               | 64.1                             | 55                                | 76.3                             |
| 55                                | 64.1                             | 65                                | 71.2                             |
| 65                                | 61.7                             | 75                                | 44.5                             |
| 75                                | 39.9                             | 85                                | 25.6                             |
| 85                                | 21.0                             | Weighted Average                  | 55.8                             |
| Weighted Average                  | 47.1                             | Weighted Average                  | 55.8                             |
Pricing GLWB - Lapses

- **Lapses**
  - Dynamic lapse function can be difficult to estimate, and tend to vary by company to company significantly.
  - What’s shown illustrates the idea but the exact form needs to be tailored and when portfolio grows a lapse study needs to be performed to refine the function.
Pricing GLWB - Mortality

- **Mortality**
  - Longevity outweighs the mortality risk
  - Base assumption already on the conservative side (80% V2000 basic)
  - Stress test with 50% of assumed

![Table 5.3.3 Comparison between base mortality and sensitivity mortality](image)
Pricing GLWB - Mortality

- **Mortality**
  - Costs increased substantially with mortality sensitivity with impact most obvious in the older ages. The much higher mortality rates at elder ages result in a life expectancy for those ages that is much more sensitive to change in mortality rates than the younger ages.
  - The expected charge level of 60 basis points is not adequate at sensitivity test. Might warrant a review of mortality study and see if the base assumption needs revised.
## Pricing GLWB – Fund Allocation

### Fund allocation

- **Base:**

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<td>15%</td>
<td>15%</td>
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- **Sensitivity:**

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<td>Aggressive Growth</td>
<td>25%</td>
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<td>20%</td>
<td>15%</td>
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<tr>
<td>Conservative Growth</td>
<td>22%</td>
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<td>3%</td>
<td>30%</td>
<td>10%</td>
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</table>
Pricing GLWB – Fund Allocation

- **Fund Allocation**
  - **Base v. Aggressive**

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<th>Assumed Fund Allocation</th>
<th>Aggressive Growth Allocation</th>
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<tr>
<td>85</td>
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</table>

- **Weighted Average**
  - **Female**: 47.1%
  - **Male**: 56.8%

With 10% more in equity funds, the cost of GLWB increases by about 20%. The expected level of 60 basis points is still adequate.
Pricing GLWB – Fund Allocation

- Fund Allocation
  - Base v. Conservative

With 15% less in equity funds, the cost of GLWB reduces by about 25%.
Pricing GLWB - Scenarios

- **Scenarios**
  - Base: historical average volatility and risk free rate
  - MC: current swap curve, 1-5 year implied volatility graded to historical average (12/31/07)

<table>
<thead>
<tr>
<th>Age</th>
<th>Female Deferral Period</th>
<th>Male Deferral Period</th>
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<td>N/A</td>
<td>N/A</td>
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</table>
Pricing GLWB - Scenarios

- Scenarios
  - The market consistent assumption would increase the cost of GLWB by up to 70% for some cells, and on aggregate the expected charge level falls short of the cost.
  - The implied volatilities have increased quite remarkably in 2007, a roughly 15% increase from September 2007 and close to a 30% increase from June 2007.
  - Costs generated with market consistent data could vary noticeably depending on when the market is calibrated to.
  - Long-term assumptions are perhaps more relevant to determination of the charge level, but the market consistent sensitivity would certainly help the insurer understand the potential risk.
Pricing GLWB - Decision

- **Decision**
  - The sensitivity tests have identified the key drivers behind the cost of GLWB.
  - All sensitivity tests, except the mortality and market consistent tests, have shown that the expected charge level of 60 basis points is adequate.
  - In reality, the actuary would then have to review these assumptions and also gather feedback from the marketing and sales team to make a final decision on the charge level.
  - For the purpose of this case study, Peter decides to go with 60 basis points. The next step is to assess the profitability of the base VA together with the embedded GLWB guarantees.
Profit Testing Entire Contract

- Profit target: 30bp of average ROA
  - ROA is ratio of present value of projected distributable earnings over present value of projected account values.
  - The distributable earnings are surpluses after tax and capital adjustments, and are discounted at the assumed required return of 12%.

- CRO also wants to know how bad profits can get

- Real World Scenarios: 1,000 pre-packaged scenarios by AAA (or user defined)

- Reserve basis: VACARVM

- Capital basis: C3P2
Profit Testing Entire Contract

- **VACARVM and C3P2 Capital projection**
  - Peter projected the VACARVM reserve and C3P2 Capital required at the end of each projection year by running a nested stochastic simulation. In other words, at each projection year along each of the 1000 real-world scenarios, Peter simulated another set of stochastic paths.
Profit Testing Entire Contract

- **VACARVM and C3P2 Capital projection**
  - Along these nested paths, account values, revenues, and expenses are projected independent of the projection along the 1000 real-world scenarios.
  - VACARVM is the 70th CTE level of the worst present value of pre-tax surplus and the C3 Phase II RBC the 90th CTE level of the worst present value of after tax surplus under these stochastic paths.
    - The CTE is the average of the results in the prescribed tail.
  - Both VACARVM and C3P2 are subject to a floor level calculated based on a “standard scenario” with economic and demographic assumptions prescribed by the AAA.
Profit Testing Entire Contract

- **VACARVM and C3P2 Capital projection**
  - The nested stochastic simulation is very time consuming.
  - The more stochastic paths generated, the more accurate the results, but the more run time.
  - Some convergence runs would need to be performed on a sample cell with reducing number of stochastic paths till the financial results start to deviate by more than desired from the base run with 1000 paths. In our model, Peter runs with 100 stochastic paths.
### Profit Testing Entire Contract

- **Average ROA**

#### Table 6.2 ROA of "US statutory" requirements

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<td>85</td>
<td>-32</td>
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**Weighted Average** 36
Profit Testing Entire Contract

- Distribution of ROA – Box Whisker

- The centerline within the box gives the median, and the up and bottom sides of the box give the 25th and 75th percentiles, respectively. The ends of the whisker give the maximum and minimum, respectively.
Profit Testing Entire Contract

- **Distribution of ROA**
  - The box is small, indicating a rather narrow range between 25th percentile and 75th percentile.
  - The upper whisker is short, indicating relatively limited up-side potential.
  - The lower whisker is long, indicating the existence of some very bad scenarios.
  - Overall, these features imply a distribution that is skewed to the left
Profit Testing Entire Contract

- Distribution of ROA – Histogram

Out of 1000 scenarios, 121 have negative ROAs and 16 have ROAs less than -100 basis points and the worst ROA is -274 basis points.
Hedge Simulation

- CRO wants to know the impact of hedging the economic reserve
  - Economic reserve, or market consistent reserve, recognizes the fair value of the GLWB as an option.
  - On economic basis, hedge helps reduce volatility in earnings.
  - On accounting basis, hedge does not.
  - However, US GAAP holds GLWB on fair value basis and hedge helps reduce GAAP earning volatility.
  - Potentially adoption of fair value reporting would make hedge more relevant to financial reporting.
Hedge Simulation

- Hedge simulation requires calculation of both economic reserve and Greeks of the GLWB in the future projection cycles.
- This again calls for nested stochastic simulation. The nested paths are risk neutral paths with risk free rates derived from the treasury yield curves from the prevailing real-world scenarios. To make the model simple, Peter has assumed that both implied volatilities and the correlation matrix will remain unchanged throughout the projection.
Hedge Simulation

- The hedge is required by CRO to be Delta/Rho hedging
  - Delta measures change in value of option as underlying asset moves
  - Rho measures change in value of option as interest rates move

- Hedge assets are Equity Futures on each of the bond index for Delta hedging, and Bond Futures on parallel Rho
Hedge Simulation

Hedge Simulation Major Steps:

- Step 1: At each projection year, summarize the hedge income or loss from the hedge position put on in the last projection cycle.
- Step 2: Calculate Delta and Rho of the economic reserve.
- Step 3: Calculate Delta of one unit of equity Futures and Rho of one unit of bond Futures.
- Step 4: Divide Delta of economic reserve by Delta of one unit of equity Futures to determine how many units of equity Futures to put on in the current projection cycle. Divide Rho of economic reserve by Rho of one unit of bond Futures to determine how many units of bond Futures to put on in the current projection cycle.
Hedge Simulation

- Hedge Results on year by year basis
  - the blue solid line captures the standard deviation of the change in economic reserve for each of the future projection year, and the red dotted line the standard deviation of the difference between change in economic reserve and hedge income.
Hedge Simulation

- **Hedge Results on year by year basis**
  - the standard deviation of the change in economic reserve before hedge in year 1 is $769, assuming a single premium of $50,000.
  - After hedge, the change in economic reserve is offset by a hedge income and the standard deviation in year 1 drops to $167, or a 78% reduction.
  - The reduction is much less toward the end of the projection, where the absolute amount of standard deviation itself reduces to a much smaller number.
Hedge Simulation

- **Hedge Results on ROA basis**

  - The distribution of the ROAs after hedge is much more centered around the mean. Its far left tails lies below that of before hedge, indicating the much fewer bad scenarios. In fact, 142 out of 1000 scenarios before hedge fall below -50 basis points whereas only 72 after hedge; 18 out of 1000 scenarios before hedge fall below -150 basis points whereas only 1 after hedge. The standard deviation of the ROAs falls by about 40% after the hedge.
Hedge Simulation

- **Caveats**
  - Hedge simulation differs from real life hedge in lots of respects
  - Projection frequency v. daily rebalancing
  - Hedge assets may also differ
  - No consideration of trading costs
  - No consideration of operational risk due to bad implementation of real life hedge
  - But still, hedge simulation helps pricing actuary understand the risk of not hedging and assess the necessity of hedging
Summary

- Peter has identified the key risk factors driving cost of GLWB
- Peter has shown that the product in its entirety meets profit target, and has assessed the probability of bad events
- Peter has shown the impact of dynamic hedging

Next: product approval, product launch, PRODUCT MANAGEMENT!!!
End