Some Simple Stochastic Models for Analyzing Investment Guarantees

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The University of Hong Kong
Introduction

- Sharp **decline** in stock markets around the world over the past few years

### Performance of World Stock Markets, 2000-2002

<table>
<thead>
<tr>
<th>Country</th>
<th>Market Index</th>
<th>Jan 2000</th>
<th>Dec 2002</th>
<th>Change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>All Ordinaries Stock Index</td>
<td>3096</td>
<td>2976</td>
<td>-3.89%</td>
</tr>
<tr>
<td>Canada</td>
<td>TSE 300 Stock Index</td>
<td>8481</td>
<td>6615</td>
<td>-22.01%</td>
</tr>
<tr>
<td>France</td>
<td>CAC 40 Stock Index</td>
<td>5660</td>
<td>3064</td>
<td>-45.87%</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>Hang Seng Stock Index</td>
<td>15532</td>
<td>9321</td>
<td>-39.99%</td>
</tr>
<tr>
<td>Japan</td>
<td>Nikkei 225 Stock Index</td>
<td>19540</td>
<td>8579</td>
<td>-56.09%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>FTSE 100 Stock Index</td>
<td>6269</td>
<td>3940</td>
<td>-37.14%</td>
</tr>
<tr>
<td>United States</td>
<td>S&amp;P 500 Stock Index</td>
<td>1394</td>
<td>880</td>
<td>-36.91%</td>
</tr>
</tbody>
</table>
Many actuaries are now being asked to employ *stochastic models* to measure solvency risk created by insurance products with equity-linked guarantees.
Introduction

- Many actuaries are now being asked to employ **stochastic models** to measure solvency risk created by insurance products with equity-linked guarantees.

- The March 2002 final report of the CIA Task Force on Segregated Fund Investment Guarantees (TFSFIG) provides useful guidance for appointed actuaries applying stochastic techniques to value segregated fund guarantees in a **Canadian** GAAP valuation environment.
In the **United States**, the Life Capital Adequacy Subcommittee (LCAS) of the American Academy of Actuaries issued the C-3 Phase II Risk-Based Capital (RBC) proposal in December 2002.
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- In addition to the interest rate risk for interest-sensitive products, the C-3 Phase II report also addresses the **equity risk exposure** inherent in variable products with guarantees, such as:
  - (1) guaranteed minimum death benefits (GMDBs);
  - (2) guaranteed minimum income benefits (GMIBs); and
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**Stochastic** scenario analysis is recommended to determine minimum capital requirements for these variable products.
In Hong Kong, the Office of the Commissioner of Insurance issued the *GN7: Guidance Note 7 on Reserving Standards for Investment Guarantees*. This note addresses:

- Class G insurance policies
- 99% level of confidence
- Scenario testing based on a stochastic model
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ASHK issued *AGN8: Process for determining liabilities under the Guidance Note 7*
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- There are a large number of potential stochastic models for equity returns. Regulators normally *do not restrict* the use of any model that *reasonably* fits the historical data.
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Reasonable $\implies$ pass a calibration test
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- Reasonable $\iff$ pass a calibration test

- The emphasis of the calibration process remains at the tails of the equity return distribution (percentiles) over different holding periods (1-5- and 10-year periods).
In this talk, we shall provide a brief review of some commonly used (in actuarial practice) stochastic investment return models:

- Independent Log-Normal models (ILN)
- Regime-Switching Log-Normal models (RSLN)
- Others:
  - Independent Log-Stable models (ILS)
  - Box-Jenkins ARIMA models
  - ARCH-type models
The log-normal model has a long and illustrious history, and has become “the workhorse of the financial asset pricing literature” (Campbell et al., 1997, p.16).

Let

\[ Y_t = \log(P_t) - \log(P_{t-1}) \]

where \( P_t \) is the end-of-period stock price (or market index), with dividends reinvested. The log-normal model assumes that log returns \( Y_t \) are independently and identically distributed (IID) normal with mean \( \mu \) and variance \( \sigma^2 \).
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At short horizons, observed returns often have negative skewness and strong evidence of excess kurtosis with time varying volatility.
The RSLN (Regime-Switching Log-Normal) model is defined as

\[ Y_t = \mu S_t + \sigma S_t \varepsilon_t \]

where \( S_t = 1, 2, \ldots, k \) denotes the unobservable state indicator which follows an ergodic \( k \)-state Markov process and \( \varepsilon_t \) is a standard normal random variable which is IID over time.

\[
Pr\{S_{t+1} = j | S_t = i\} = p_{ij}, \quad 0 \leq p_{ij} \leq 1, \quad \& \quad \sum_{j=1}^{k} p_{ij} = 1 \quad \text{for all } i.
\]
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The stochastic transition probabilities that determine the evolution in \( S_t \) (so that the states follow a homogenous Markov chain) is given by

\[ Pr\{S_{t+1} = j|S_t = i\} = p_{ij}, \quad 0 \leq p_{ij} \leq 1, \quad \text{and} \quad \sum_{j=1}^{k} p_{ij} = 1 \quad \text{for all } i. \]
RSLN Models

- In most situations, \( k = 2 \) or \( k = 3 \) (that is two- or three-regime models) is sufficient for modelling monthly equity returns (Hardy, *NAAJ*, 2001)
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The use of RSLN processes for modelling maturity guarantees has been gaining popularity

Both the Canadian Institute of Actuaries (CIA) and the American Academy of Actuaries (AAA) used a two-regime RSLN model for developing the calibration test
A simple two-regime RSLN model:

\[ Y_t = \begin{cases} 
\varepsilon_t^{(1)} & \sim N(\mu_1, \sigma_1^2), \\
\varepsilon_t^{(2)} & \sim N(\mu_2, \sigma_2^2),
\end{cases} \]

with transition probability matrix

\[ \mathcal{P} = \begin{pmatrix} p_{11} & p_{12} \\
p_{21} & p_{22} \end{pmatrix}, \quad 0 < p_{ij} < 1. \]  

This implies that the vector of steady-state (ergodic) probabilities is

\[ \begin{pmatrix} \pi_1 \\
\pi_2 \end{pmatrix} = \begin{pmatrix} \frac{p_{21}}{p_{12} + p_{21}} \\
\frac{p_{12}}{p_{12} + p_{21}} \end{pmatrix}. \]
Log-Stable Models

- Stable distributions are a class of probability laws that are able to accommodate heavy tails and skewness, which are frequently seen in investment data.

\[ Y_t \sim S_0(\alpha, \beta, \delta, \mu) \]

The characteristic function of a stable distribution is given by:

\[
\phi(t) = \exp \left( \begin{array}{c}
\alpha \beta t \sin(\delta t) & \kappa \\
0 & \kappa \end{array} \right) + \int_{-\infty}^{\infty} \exp \left( i \beta u \sin(\delta u) - \frac{1}{2} \kappa^2 |u|^\alpha \right) f(u) du
\]

It should be noted that Gaussian distributions are special cases of stable laws with \( \alpha = 2 \) and \( \beta = 0 \); more precisely, \( N(\mu, \sigma^2) \equiv S_0(2, 0; = \sqrt{2\pi}) \).
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- Stable distributions are a class of probability laws that are able to accommodate heavy tails and skewness, which are frequently seen in investment data.

- A random variable $Y_t$ is $S_0(\alpha, \beta, \gamma, \delta)$, if its characteristic function takes the form:

$$
\Psi(t) = E[\exp(itY)] = \begin{cases} 
\exp\left(-\gamma^\alpha|t|^{\alpha} \left[1 + i\beta \text{sign}(t) \left(\tan \frac{\pi \alpha}{2}\right) (|t|^{1-\alpha} - 1)\right] + i\delta t\right), & \alpha \neq 1, \\
\exp\left(-\gamma|t| \left[1 + i\beta \text{sign}(t) \left(\frac{2}{\pi}\right) (\ln(|t|))\right] + i\delta t\right), & \alpha = 1.
\end{cases}
$$

It should be noted that Gaussian distributions are special cases of stable laws with $\alpha = 2$ and $\delta = 0$; more precisely, $N(\mu, \sigma^2) = S_0(2; 0, \sigma^2) = \sqrt{2\pi} \sigma$. Some Simple Stochastic Models for Analyzing Investment Guarantees – p. 13/36
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\exp\left(-\gamma|t| \left[1 + i\beta \text{sign}(t) \left(\frac{2}{\pi} \right) \left(\ln(|\gamma t|)\right)\right] + i\delta t\right), & \alpha = 1.
\end{cases}
\]

It should be noted that Gaussian distributions are special cases of stable laws with \( \alpha = 2 \) and \( \beta = 0 \); more precisely, \( N(\mu, \sigma^2) = S0(2, 0, \sigma/\sqrt{2}, \mu) \).
ARMA Models

- ARMA (autoregressive moving average) or called Box-Jenkins models
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ARMA (autoregressive moving average) or called Box-Jenkins models


An ARMA\((p, q)\) model:

\[
\phi(L)Y_t = \theta(L)\alpha_t,
\]

where \(L\) is the backshift operator such that \(L^s Y_t = Y_{t-s}\),

\[
\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p, \quad \theta(L) = \theta_1 L - \ldots - \theta_q L^q.
\]
ARCH Models

Robert Engle, who proposed this class of models in 1982, won the 2003 Nobel Prize in Economic Sciences
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- Autoregressive conditional heteroscedastic (ARCH) models
  - Engle (1982): ARCH
  - Bollerslev (1986): GARCH

Useful because these models are able to capture empirical regularities of asset returns such as thick tails of unconditional distributions, volatility clustering and negative correlation between lagged returns and conditional variance.
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ARCH Models

- Let $a_t = (Y_t - \mu)$ be the mean-corrected log return.
Let \( a_t = (Y_t - \mu) \) be the mean-corrected log return.

In this talk, we consider a GARCH(1,1) process and its innovation following a Student \( t \) distribution:

\[
\varepsilon_t = a_t / \sqrt{h_t},
\]

and \( \varepsilon_t \) in the above equation has a marginal \( t \) distribution with mean zero, unit variance and degrees of freedom \( \nu \), and the conditional variance has the following representation

\[
h_t = \omega + \beta h_{t-1} + \alpha a_{t-1}^2.
\]
The final report of the CIA Task Force on Segregated Fund Investment Guarantees states, in its Appendix D, that *the model should be fitted using maximum likelihood estimation (MLE) or a similar statistical procedure*. 
The MLE of the stochastic models discussed in this talk can be obtained by:

- **Independent Log-Normal models (ILN)**
  - EXCEL
- **Regime-Switching Log-Normal models (RSLN)**
  - EXCEL, freeware from SoA
- **Independent Log-Stable models (ILS)**
  - VB, freeware from Nolan
- **Box-Jenkins ARMA models**
  - SAS & Many others
- **ARCH-type models**
  - GAUSS & Many others
Calibration Tests

Even though there is no mandatory class of stochastic models for fitting the baseline data, it is recommended that the final model be calibrated to some specified tail distribution percentiles of the accumulation factors

\[ A_m = \frac{P_m}{P_0} \]
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- Three different holding periods: 1 year, 5 years and 10 years
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- Three different holding periods: 1 year, 5 years and 10 years

- The CIA report (TFSFIG, 2002) recommends a set of model calibration points based on TSE 300 monthly total return series (the benchmark series), from January 1957 to December 1999.
Calibration Tests

In the United States, the AAA uses an approach similar to the CIA, where a regime-switching log-normal model is fitted to monthly total return data (Benchmark series: S&P 500 total returns), January 1945 to October 2002.
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Calibration Tests

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- Three different holding periods: 1 year, 5 years and 10 years

- A set of 30 accumulated wealth calibration points that must be materially met by the equity model used to determine capital requirements
### Exhibit 2
The Calibration Points for C-3 Equity Return Models

<table>
<thead>
<tr>
<th>Calibration Point (%)</th>
<th>1 Year</th>
<th>5 Year</th>
<th>10 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.65</td>
<td>0.58</td>
<td>0.67</td>
</tr>
<tr>
<td>1.0</td>
<td>0.70</td>
<td>0.66</td>
<td>0.79</td>
</tr>
<tr>
<td>2.5</td>
<td>0.77</td>
<td>0.78</td>
<td>1.00</td>
</tr>
<tr>
<td>5.0</td>
<td>0.84</td>
<td>0.91</td>
<td>1.21</td>
</tr>
<tr>
<td>10.0</td>
<td>0.91</td>
<td>1.07</td>
<td>1.51</td>
</tr>
<tr>
<td>90.0</td>
<td>1.35</td>
<td>2.73</td>
<td>5.79</td>
</tr>
<tr>
<td>95.0</td>
<td>1.42</td>
<td>3.07</td>
<td>6.86</td>
</tr>
<tr>
<td>97.5</td>
<td>1.48</td>
<td>3.39</td>
<td>7.94</td>
</tr>
<tr>
<td>99.0</td>
<td>1.55</td>
<td>3.79</td>
<td>9.37</td>
</tr>
<tr>
<td>99.5</td>
<td>1.60</td>
<td>4.10</td>
<td>10.48</td>
</tr>
</tbody>
</table>

Note: Calibration Points Are Based on S&P Total Return Accumulation Factors.
CIA Calibration Points

Note: Canadian Calibration Points are based on TSE 300 Total Return Accumulation Factors

<table>
<thead>
<tr>
<th>Accumulation period</th>
<th>2.5&lt;sup&gt;th&lt;/sup&gt; percentile</th>
<th>5&lt;sup&gt;th&lt;/sup&gt; percentile</th>
<th>10&lt;sup&gt;th&lt;/sup&gt; percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>One year</td>
<td>0.76</td>
<td>0.82</td>
<td>0.90</td>
</tr>
<tr>
<td>Five years</td>
<td>0.75</td>
<td>0.85</td>
<td>1.05</td>
</tr>
<tr>
<td>Ten years</td>
<td>0.85</td>
<td>1.05</td>
<td>1.35</td>
</tr>
</tbody>
</table>
In the second part of this talk, we shall attempt to develop a set of calibration points for the Hong Kong market.
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- Hang Seng total return historical experience as a proxy for returns on a broadly diversified Hong Kong equity fund.
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Hang Seng total return historical experience as a proxy for returns on a broadly diversified Hong Kong equity fund.

Hence, monthly Hang Seng Index (HSI) total return series, from June 1973 to September 2003 ($n = 363$), is chosen as the benchmark series in this study.
HSI Total Return

Fitted Models

- Maximum Likelihood Estimation (MLE)
Fitted Models

- Maximum Likelihood Estimation (MLE)
- ILN

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>μ</td>
<td>0.0104548</td>
</tr>
<tr>
<td>σ</td>
<td>0.0983899</td>
</tr>
</tbody>
</table>

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Fitted Models

- Maximum Likelihood Estimation (MLE)
- ILN

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>$\mu$</td>
<td>0.0104548</td>
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</tr>
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<td>$\sigma$</td>
<td>0.0983899</td>
<td></td>
</tr>
</tbody>
</table>

- RSLN

<table>
<thead>
<tr>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1 = 0.021704928$</td>
<td>$\mu_2 = -0.023300785$</td>
</tr>
<tr>
<td>$\sigma_1 = 0.065898426$</td>
<td>$\sigma_2 = 0.155492034$</td>
</tr>
<tr>
<td>$p_{11} = 0.95011$</td>
<td>$p_{22} = 0.85492$</td>
</tr>
<tr>
<td>$p_{12} = 0.04989$</td>
<td>$p_{21} = 0.14508$</td>
</tr>
<tr>
<td>$\pi_1 = 71%$</td>
<td>$\pi_2 = 26%$</td>
</tr>
</tbody>
</table>
Fitted Models

- Other models
Fitted Models

- Other models
- ILS

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>1.7028</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.3357</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0563</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1903</td>
</tr>
</tbody>
</table>
Fitted Models

- Other models
- ILS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>0.1903</td>
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</tbody>
</table>

- GARCH(1,1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.01550</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.00997</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.00005</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.00000</td>
</tr>
<tr>
<td>$\nu$</td>
<td>3.96</td>
</tr>
</tbody>
</table>
Both the CIA and AAA employed a two-regime RSLN model for their baseline series.
The Final Model

- Both the CIA and AAA employed a two-regime RSLN model for their baseline series.
- The HSI total returns are reasonably fitted by a two-regime RSLN process.
The Final Model

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- The HSI total returns are reasonably fitted by a two-regime RSLN process.

- Interpretation of the RSLN2 model:
  - Regime 1 – Usual, Stable Normal-Volatility State
  - Regime 2 – Occasional, Unstable High-Volatility State
The Final Model

Density Function for Monthly Return, Hang Seng Index
Calibration points can be developed based on the final fitted RSLN model.

<table>
<thead>
<tr>
<th>Quantile (%)</th>
<th>1-Year</th>
<th>5-Year</th>
<th>10-Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.37</td>
<td>0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>2.5</td>
<td>0.94</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>5.0</td>
<td>0.98</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>10.0</td>
<td>1.02</td>
<td>0.58</td>
<td>0.67</td>
</tr>
<tr>
<td>90.0</td>
<td>1.37</td>
<td>5.28</td>
<td>15.70</td>
</tr>
<tr>
<td>95.0</td>
<td>1.43</td>
<td>6.83</td>
<td>22.85</td>
</tr>
<tr>
<td>97.5</td>
<td>1.49</td>
<td>8.45</td>
<td>31.50</td>
</tr>
<tr>
<td>99.0</td>
<td>2.38</td>
<td>10.83</td>
<td>45.26</td>
</tr>
</tbody>
</table>
AAA C-3 Phase I — 95th percentile standard
CTE

- **AAA C-3 Phase I** — 95th percentile standard

- **AAA C-3 Phase II** — proposed RBC requirement would be based on a modified *Conditional Tail Expectation* (CTE) measure. The AAA report can be found at:

  www.actuary.org/pdf/life/rbc_16dec02.pdf
CTE

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What is CTE? How to compute CTE for RSLN generated scenarios?
Some Simple Stochastic Models for Analyzing Investment Guarantees – p. 31/36
Risk Measures for maturity guarantee liability under equity-linked contracts:

- \( \xi \) – the probability that the guarantee will not be exercised
- \( Q_\alpha \) – quantile risk measure (value-at-risk) of the liability
- \( \text{CTE}(\alpha) = E[X | X > Q_\alpha] \)
Applications

- Risk Measures for maturity guarantee liability under equity-linked contracts:
  - $\xi$ – the probability that the guarantee will not be exercised
  - $Q_\alpha$ – quantile risk measure (value-at-risk) of the liability
  - $\text{CTE}(\alpha) = E[X \mid X > Q_\alpha]$

- Given a stochastic equity model and details of the contracts, these measures can be computed analytically or by simulations.
An Example

- We adopt a simple example from Hardy (2001, *NAAJ*): A segregated fund contract that matures in 10 years; the guarantee is *equal* to the fund market value at the start of the projection. Management fees of 0.25% per month, compounded continuously, are deducted. Lapses and deaths are ignored for simplicity.
An Example

- $\xi$, $Q_\alpha$ and $CTE(\alpha)$ are computed using fitted RSLN models for the U.S. (S&P500), Canadian (TSE300) and Hong Kong (HSI) markets.

<table>
<thead>
<tr>
<th>Markets</th>
<th>HSI</th>
<th>TSE300</th>
<th>S&amp;P500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>0.7808</td>
<td>0.8827</td>
<td>0.9572</td>
</tr>
<tr>
<td>$Q_{0.900}$</td>
<td>50.63</td>
<td>5.81</td>
<td>0.00</td>
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<tr>
<td>$Q_{0.950}$</td>
<td>71.01</td>
<td>25.95</td>
<td>0.00</td>
</tr>
<tr>
<td>$Q_{0.975}$</td>
<td>82.04</td>
<td>40.44</td>
<td>12.41</td>
</tr>
<tr>
<td>CTE(0.900)</td>
<td>71.71</td>
<td>29.22</td>
<td>0.03</td>
</tr>
<tr>
<td>CTE(0.950)</td>
<td>82.67</td>
<td>43.13</td>
<td>15.88</td>
</tr>
<tr>
<td>CTE(0.975)</td>
<td>88.93</td>
<td>53.53</td>
<td>28.17</td>
</tr>
</tbody>
</table>

Note: $Q_\alpha$ and $CTE(\alpha)$ are expressed as percentage of the fund value.
A Scenario Generator
Thank You!